QCD and Supersymmetry

Lecture 1: What do we know about $\mathcal{N} = 1$ Super Yang-Mills ?

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Motivation

The most interesting regime of QCD where chiral symmetry breaking and confinement occurs is non-perturbative. Unfortunately, it is difficult to derive analytical results in this regime.

In supersymmetric theories the situation is much better (due to holomorphicity): we can derive exact non-perturbative results in $\mathcal{N} = 1$ Super Yang-Mills and in Super QCD.

It would be fantastic if we could use the knowledge that we accumulated in supersymmetric QCD and use it for real QCD !

Motivation - continuation

In these lectures I will present a method, called *"planar equivalence"*, which enables us to copy results from supersymmetric theories to QCD.

I have decided to organize the lectures as follows:

Lecture number 1: what do we know about $\mathcal{N} = 1$ SYM ?

Lecture number 2: What is orientifold planar equivalence ?

Lecture number 3: Supersymmetry relics in QCD

The $\mathcal{N} = 1$ Super Yang-Mills Lagrangian

$\mathcal{N} = 1$ Super Yang-Mills is a theory which consists of a gluon and a gluino. The gluino is the "superpartner" of the gluon. It is a four-component Majorana (real) fermion transforming in the adjoint representation of the gauge group. We will focus on $SU(N)$.

The Lagrangian of $\mathcal{N} = 1$ SYM is

$$\mathcal{L} = -\frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{i}{2g^2} \bar{\lambda}^a D_\mu \gamma^\mu \lambda^a$$

The Lagrangian looks exactly as the Lagrangian of QCD with one quark flavor, except that in the present case the quark field (denoted by λ) transforms in the *adjoint* representation.

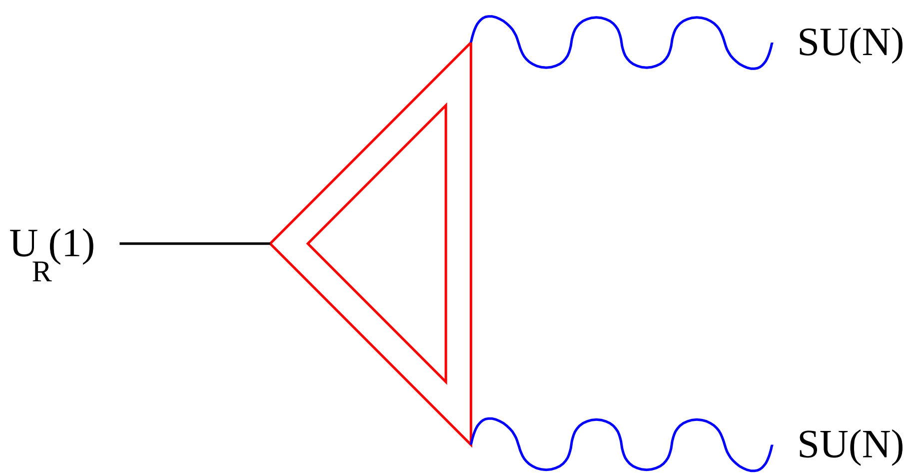
The theory is expected to confine and to have a mass-gap of degenerate color-singlets ("glueballs" and "glueballinos").

$U_R(1)$ and the axial anomaly

The SYM theory admits, at the classical level, a $U(1)$ axial symmetry (similar to the $U_A(1)$ in QCD), often called “R-symmetry”, of the form (using the Weyl notation)

$$\lambda^\alpha \rightarrow e^{i\beta}\lambda^\alpha$$

The associated Noether current is $J_\mu = \frac{1}{g^2}\bar{\lambda}\gamma_\mu\gamma_5\lambda$. The $U_R(1)$ symmetry is broken by the triangle anomaly



$$\partial^\mu J_\mu = \frac{N}{16\pi^2} F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}$$

The $U_R(1)$ anomaly

Notice that the anomaly is proportional to N, since the gluino transforms in the adjoint representation and $\mathrm{tr}\, T^a T^b = N\delta^{ab}$ (in contrast to QCD where the quarks are in the fundamental and the axial anomaly is *not* proportional to N, since $\mathrm{tr}\, t^a t^b = \frac{1}{2}\delta^{ab}$).

Thus,

$$U_R(1) \rightarrow Z_{2N}$$

Namely, a residual transformation of the form

$$\lambda \rightarrow e^{i\pi\frac{k}{N}}\lambda$$

leaves the partition function invariant.

The fate of the Z_{2N} symmetry

Witten conjectured in 1982 (Witten, 1982) that the Z_{2N} symmetry is spontaneously broken down to Z_2.

That indeed happens and the result is a theory with N vacua, where the gluino condensate serves as the order parameter of the breaking, namely

$$\langle \lambda^\alpha \lambda^\alpha \rangle_k = \text{const.}\, \Lambda^3 e^{i2\pi \frac{k}{N}}$$

where $k = 0, ..., N-1$ labels the various vacua.

How do we know that a gluino condensate is formed and that $Z_{2N} \to Z_2$?

An evidence for the existence of a gluino condensate follows from the Veneziano-Yankielowicz effective action.

The Veneziano-Yankielowicz effective action

In 1982, from anomaly considerations, Veneziano and Yankielowicz wrote an effective action for $\mathcal{N} = 1$ SYM, (Veneziano, Yankielowicz, 1982). The fundamental (super)-field that appears in their Lagrangian is $S = \lambda\lambda$. The Lagrangian for S is

$$I = N(S \log S/\Lambda^3 - S)\,.$$

The equation of motion for S is

$$N \log S/\Lambda^3 = c\,.$$

Hence

$$(S/\Lambda^3)^N = c'\,,$$

or

$$\langle S \rangle = \langle \lambda\lambda \rangle = c'' \Lambda^3 e^{i2\pi \frac{k}{N}}\,.$$

The Veneziano-Yankielowicz Lagrangian does not fix the value of the gluino condensate, it only demonstrates its non-vanishing.

In order to fix the value of the gluino condensate a calculation is needed.

The gluino condensate

There are various methods of calculating the gluino condensate. (Shifman and Vainshtein, 1988), (Seiberg and Witten, 1994), (Davies, Hollowood, Khoze and Mattis, 1999)

All the methods rely on holomorphy. I prefer the last method where the authors compactified the theory on a small circle to make the theory weakly coupled. They calculated the gluino condensate by showing that $\langle\lambda\lambda\rangle$ is saturated solely by monopoles. Then they showed that the value does not depend on the radius of the circle, due to holomorphicity. So, they could de-compactify the theory to obtain

$$\langle\lambda\lambda\rangle_k = -6N\Lambda^3 e^{i2\pi\frac{k}{N}} \ .$$

The gluino condensate is analogous to the quark condensate $\langle\bar{q}q\rangle$ in QCD. The calculation of such a quantity is a great achievement !

The NSVZ beta function

By analyzing the response of the coupling to the variation of the UV-cutoff in the expression for the gluino condensate, NSVZ obtained an important result: the *exact* beta function (Novikov, Shifman, Vainshtein and Zakharov, 1983)

$$\beta(g) = -\frac{g^3}{16\pi^2}\frac{3N}{1-\frac{g^2 N}{8\pi^2}}$$

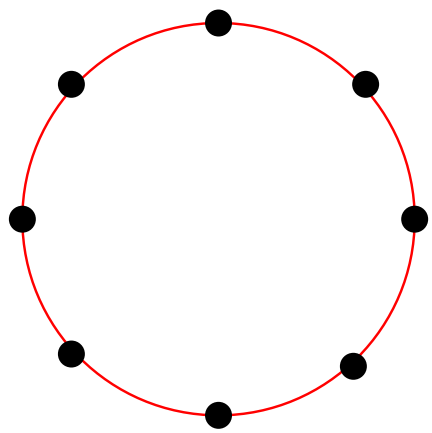
The NSVZ beta function is exact to all orders in perturbation theory. The first two coefficients are universal (namely scheme independent) and they match the explicit one and two loop calculations.

The generalized NSVZ beta function for super-QCD played a central role in fixing the conformal window in SQCD and in the derivation of Seiberg duality (Seiberg, 1994).

Intermediate summary

So far, we learnt that $\mathcal{N} = 1$ SYM has an interesting and rich structure.

The vacuum structure of the theory is depicted in the figure below



The Vacuum Structure of
SU(8) N=1 Super Yang–Mill

We know several exact results about the theory:

i. There are N degenerate vacua.

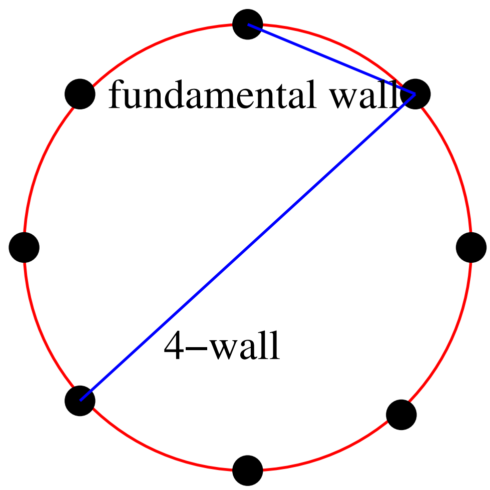
ii. A gluino condensate $\langle \lambda\lambda \rangle$ is formed and we know its exact value.

iii. The all orders (NSVZ) beta function is also known.

What else do we know about $\mathcal{N} = 1$ SYM ?

Domain Walls

When a discrete symmetry, such as Z_{2N} is spontaneously broken, there exists domain walls which interpolate between the various vacua of the theory, see the figure below



Domain Walls in SU(8)
N=1 Super Yang–Mills

The domain walls are (2+1) dimensional objects, localized at, say $x_3 = z$. The "fundamental" wall interpolates between neighbouring vacua, whereas a k-wall "skips" k vacua, as depicted in the figure.

We can think about the k-wall as a bound state of k elementary walls.

Domain Walls Tension

The tension of a k-wall is given by (Dvali and Shifman, 1996) the difference between the values of the gluino condensate in the given vacua

$$T_k = \frac{N}{8\pi^2}|\langle\lambda\lambda\rangle_{l+k} - \langle\lambda\lambda\rangle_l| = \frac{3N^2}{2\pi^2}\Lambda^3 \sin(\pi\frac{k}{N})$$

Note that when k is kept fixed and $N \to \infty$, $T_k \sim kT_1$ and also $T_1 \sim N\Lambda^3$.

It means that in the large-N limit the k-wall becomes a collection of k non-interacting fundamental domain walls.

The tension of each fundamental domain wall is proportional to N. This is surprising since solitons in a theory with adjoint matter should carry a tension $\sim N^2$.

In fact, every quantity (at least in perturbation theory, or semiclassically) should depend on N^2. This observation led to a bold conjecture by Witten ...

Domain Walls as D-branes

In 1998 Witten argued that the $\mathcal{N} = 1$ SYM domain walls are *QCD D-branes*.

What does it mean ?

Usually we think about (a large-N) gauge theory as a string theory of closed and open strings. The closed strings are identified with flavor singlets - the glueballs of the theory, whereas the open strings are identified with mesons.

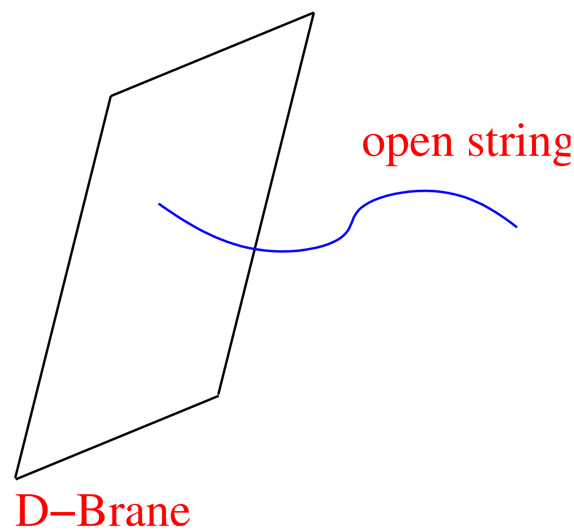
But now we know that in string theory there exists extended objects which are called "branes". Thus, if $\mathcal{N} = 1$ SYM is described by (or dual to a) string theory, where are the D-branes in field theory ? Witten argue that these are the domain walls.

Domain Walls as D-branes

In string theory D-branes carry the following character

i. They are non-perturbative objects. Their tension is $T \sim \frac{1}{g_{string}}$.

ii. An open string can end on a D-brane.



iii. D-branes interact by an exchange of closed strings.

Domain Walls as D-branes

Witten noticed that the tension of the fundamental domain wall is $T_1 \sim N$ (at large-N). By using the relation between field theory and string theory $g_{string} \sim \frac{1}{N}$, we observe that the domain wall tension matches the expectation from a D-brane.

Moreover, Witten argued that the QCD-string (flux tube) can end on the domain wall, in the same way that the open-string ends on a D-brane.

Finally, together with Shifman (Armoni and Shifman, 2003) we showed how domain wall interact via an exchange of glueballs. In fact, at large-N there is a cancellation between the attraction due to an exchange of even-parity glueballs and the repulsion due to an exchange of odd-parity glueballs.

This is similar to the cancellation of the interaction between parallel D-branes (dilaton and graviton against RR-fields).

The Acharya-Vafa theory

It is well known in string theory, that when Dp-branes coincide there appears a $(p+1)$ dimensional field theory "on the branes".

If domain walls are the "QCD D-branes", does something similar happen on the domain wall ?

It was argued by Acharya and Vafa that this is indeed the case (Acharya and Vafa, 2001). The field theory on the k wall was argued to be a $U(k)$ gauge theory which contains a level N Chern-Simons term

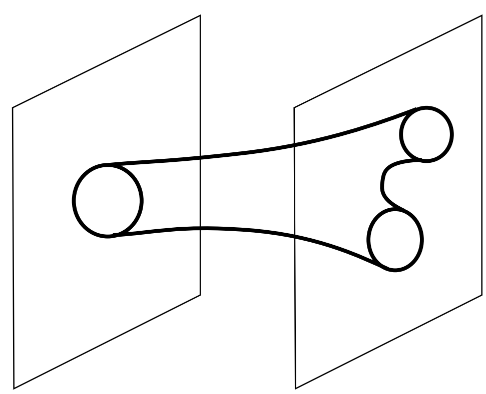
$$\mathcal{L}_{\text{Acharya}-\text{Vafa}} = \frac{1}{2g^2}\mathrm{tr}\left(-\frac{1}{2}F_{mn}^2 - (D_i\phi)^2 \right.$$
$$\left. + N\epsilon_{ijk}(A^i\partial^j A^k + \frac{1}{3}A^i A^j A^k) + \text{fermions}\right)$$

The Potential between Domain-Walls

By using the Acharya-Vafa theory, it is possible to calculate the force between domain walls (Armoni and Hollowood, 2005, 2006).

As in standard D-branes physics, the vev of the scalar, $x \equiv \langle \phi \rangle$, parametrizes the distance between domain walls.

A Coleman-Weinberg effective potential for ϕ, is interpreted as the potential between a pair of parallel domain walls



It turns out that the one-loop effective potential vanishes and we had to perform a two-loops calculation. The result is

$$V(x) \sim \frac{1}{N} \frac{x^2}{1+x^2} \, .$$

Summary

Today we learnt quite a lot about $\mathcal{N}=1$ SYM

i. The theory posses N degenerate vacua.

ii. Gluino condensation is formed
$\langle\lambda\lambda\rangle_k = -6N\Lambda^3 e^{i2\pi\frac{k}{N}}$.

iii. $\beta_{\rm NSVZ}(g) = -\frac{g^3}{16\pi^2}\frac{3N}{1-\frac{g^2N}{8\pi^2}}$.

iv. There exists domain walls whose tension is
$T_k = \frac{N}{8\pi^2}|\langle\lambda\lambda\rangle_{l+k} - \langle\lambda\lambda\rangle_l| = \frac{3N^2}{2\pi^2}\Lambda^3\sin(\pi\frac{k}{N})$.

v. Domain walls are "QCD D-branes".

Tomorrow we will learn one more thing about $\mathcal{N}=1$ SYM:

We can use some of the above results for QCD !