QCD and Supersymmetry

Lecture 3: Supersymmetry Relics in QCD

Adi Armoni

Swansea University

Schladming 2009

Motivation

In the first two lectures we learnt about the non-perturbative properties of $\mathcal{N} = 1$ SYM and that a certain "orientifold gauge theory" becomes equivalent at large-N to $\mathcal{N} = 1$ SYM.

For $SU(3)$ the "orientifold" theory becomes ordinary one-flavor QCD, so we can use the connection with $\mathcal{N} = 1$ SYM to estimate quantities in QCD.

SUSY Relics in the QCD Spectrum

In supersymmetric theories the color-singlets spectrum is bose/fermi degenerate. Moreover, massive color-singlet bosons are also parity even/odd degenerate.

In the large-N orientifold theory there are only bosonic color-singlets, since it is impossible to form a "light" fermionic color-singlet from antisymmetric constituents.

We thus predict that in one flavor QCD the color singlets are approximately even/odd parity degenerate (Armoni, Shifman and Veneziano, 2003).

Later, together with Imeroni (Armoni and Imeroni (2005), we argued by using some insight from type 0' string theory that the ratio between the η' and the σ masses is given by the ratio of the axial and conformal anomalies

$$M_{\eta'}/M_\sigma \sim (N-2)/N$$

Hence for one-flavor QCD $M_{\eta'}/M_\sigma \sim 1/3$!

SUSY Relics in the QCD Spectrum

The prediction that the η' is much lighter than the σ is rather surprising. In real QCD the η' is heavier than the σ (although the σ is broad. This is not the case in one-flavor QCD).

In order to check this prediction, a lattice simulation of one-flavor QCD was carried out by the Munster collaboration (Farchioni et.al. 2007). Their result for the masses of η' and σ, extrapolated to the chiral limit, are

$$M_{\eta'}/M_{\sigma} = 0.410(32)(25)\,,$$

In a reasonable agreement with our estimate.

The QCD quark condensate from SUSY

The QCD quark condensate $\langle \bar{q}q \rangle$ is the order parameter for chiral symmetry breaking. It is a non-perturbative quantity and hence very difficult to compute. The only reliable method of calculating it is the lattice.

Since we know the value of the gluino condensate, we can use it to estimate the value of the quark condensate.

Let us define a renormalization group invariant (RGI) quantity

$$(g^2)^{\gamma/\beta}\, \bar{\Psi}_{[ij]}\Psi^{[ij]}\,.$$

The above quantity coincides with the gluino condensate when $N \to \infty$.

In order to estimate the (large !) $1/N$ corrections, notice that for the orientifold $SU(2)$ theory the quark condensate vanishes, since for $SU(2)$ the antisymmetric and the singlet are the same representation.

The QCD quark condensate from SUSY

We thus argue that

$$(g^2)^{\gamma/\beta}\bar{\Psi}_{[ij]}\Psi^{[ij]} = -6(N-2)\Lambda^3_{\overline{\mathrm{MS}}}\mathcal{K}(1/N)\,,$$

where $\mathcal{K}(1/N) = 1 + O(1/N)$.

In order to compare our estimate with lattice results, which are often quoted at a scale 2 GeV, we evaluated the quark condensate at that scale.

Our result (including 30% due to $1/N$ corrections) is

$$\langle\bar{q}q\rangle_{2\,\mathrm{GeV}} = -(270 \pm 30\,\mathrm{MeV})^3\,.$$

This number has to be compared with the results of (DeGrand et.al. 2006)

$$\langle\bar{q}q\rangle_{2\,\mathrm{GeV}} = -(269(9)\,\mathrm{MeV})^3\,.$$

A lattice check of planar equivalence

It fairly easy to check some predictions of planar equivalence by using a lattice simulation of the *quenched theory* (Armoni, Lucini, Patella and Pica, 2008)

$$\langle \bar{\psi}\psi \rangle = \int DA_\mu (\exp iS_{\mathrm{YM}})\, \mathrm{tr} \frac{1}{i\gamma_\mu D_\mu - m}$$

Planar equivalence implies

$$\lim_{N\to\infty} \left[\frac{\langle \lambda\lambda \rangle_{\mathrm{Adj}}}{N^2} - \frac{\langle \bar{\psi}\psi \rangle_{\mathrm{Sym}} + \langle \bar{\psi}\psi \rangle_{\mathrm{Asym}}}{2N^2} \right] = 0$$

Let us parametrise the condensate as follows

$$\frac{\langle \bar{\psi}\psi \rangle_{\mathrm{Sym/Asym}}}{N^2} = f\left(\frac{1}{N^2}, m\right) \pm \frac{1}{N} g\left(\frac{1}{N^2}, m\right)$$

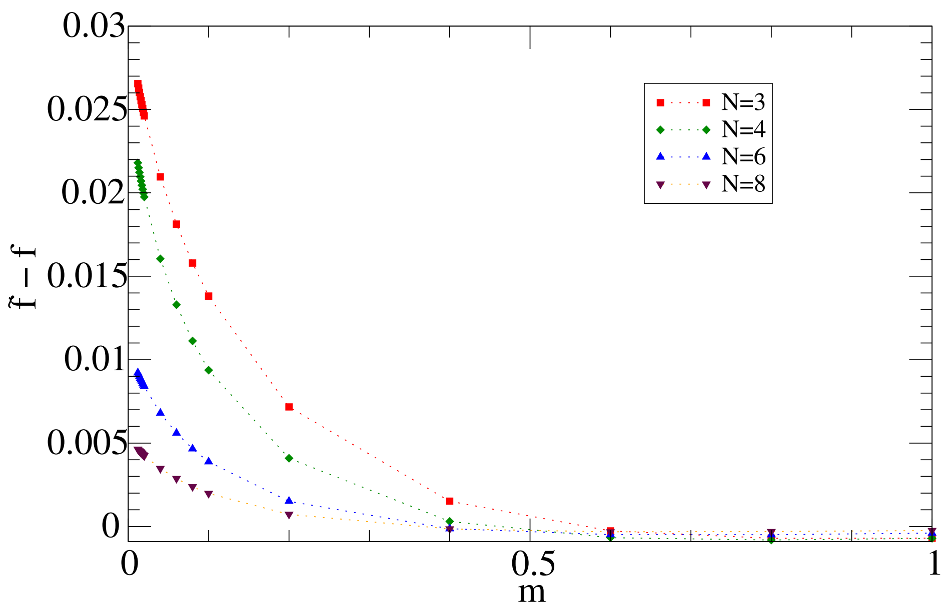
$$\frac{\langle \lambda\lambda \rangle_{\mathrm{Adj}}}{N^2} = \tilde{f}\left(\frac{1}{N^2}, m\right) - \frac{1}{2N^2} \langle \bar{\psi}\psi \rangle_{\mathrm{free}}$$

A lattice check of planar equivalence

We expect

$$f\left(\frac{1}{N^2}, m\right) - \tilde{f}\left(\frac{1}{N^2}, m\right) = \frac{c_1(m)}{N^2} + \frac{c_2(m)}{N^4} \ .$$

The lattice simulation was conducted for $SU(2)$, $SU(3)$, $SU(4)$, $SU(6)$ and $SU(8)$, for various values of masses. The results are presented in the plot below



Three-flavors QCD

It is possible to generalize (Armoni, Shore and Veneziano, 2005) the analysis to three-flavors QCD.

The idea is to consider an $SU(N)$ theory with one-flavor in the antisymmetric representation and two additional flavors in the fundamental representation.

This model reduces to three flavors QCD for $SU(3)$, becomes SUSY in the bosonic sector in the large N limit, but differs from the previous model by the $1/N$ corrections. The result for the quark condensate as a function of the 't Hooft coupling $\lambda \equiv \frac{g^2 N}{2\pi}$, at $2\,\mathrm{GeV}$ is

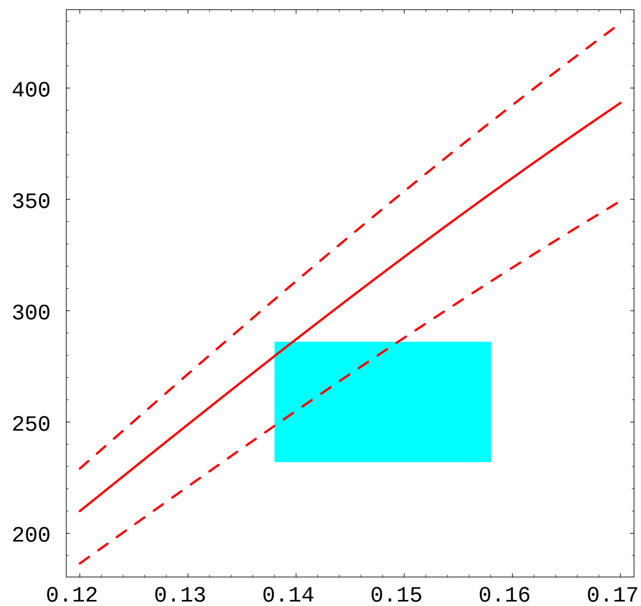
$$\langle \bar{q} q \rangle_{2\,\mathrm{GeV}} = -\frac{12}{\pi^2} \lambda^{-\frac{44}{27}} e^{-\frac{1}{\lambda}}$$

Assuming (taken from the PDG) $\lambda(2\,\mathrm{GeV}) = 0.148 \pm 0.010$, we obtain ...

Three-flavors QCD

$$\langle \bar{q}q \rangle_{2\,\mathrm{GeV}} = -(317 \pm 30 \pm 36\ \mathrm{MeV})^3$$

The two errors are ±30% due to $1/N$ corrections and the experimental uncertainty of λ at 2 GeV.



The above plot is the quark condensate in three flavor QCD, expressed as $-(y\,\mathrm{MeV})^3$, as a function of the 't Hooft coupling at 2 GeV.

The blue area represents the lattice data (McNeile, 2005). The solid line is the prediction of the quark condensate. The two dashed lines represent the $1/N$ (30%) error.

NSVZ beta function for QCD

Another prediction of planar equivalence is an NSVZ beta function for one-flavor QCD

$$\beta(g) = -\frac{g^3}{16\pi^2}\frac{3N}{1-\frac{g^2 N}{8\pi^2}}$$

Of course the above expression is only an approximation and it suffers from $1/N$ corrections.

As we shall see, the NSVZ beta function will be used to support the existence of a Seiberg duality in a non-supersymmetric theory.

Supersymmetry between Mesons and Baryons

It is well known that both mesons and baryons admit a linear Regge trajectory of the form

$$J = \alpha_0 + \alpha' M^2 \,,$$

with an almost identical Regge slope $\alpha' \sim 0.89\,\mathrm{GeV}^{-2}$.

The coincidence of the Regge slopes is explained by the diquark picture. The meson consists of a quark anti-quark connected by a string and similarly the baryon consists of a quark diquark connected by a string.

Let us propose a different explanation (Armoni and Patella, 2009). In our model the meson is a standard quark anti-quark connected by a string.

$$M = \bar{Q}^j(x_0) \left(P \exp i \int_{x_0}^{x_1} \vec{A}(y) d\vec{y} \right)_j^i Q_i(x_1)$$

Supersymmetry between Mesons and Baryons

The baryon is made of two quarks which are connected to a smeared antisymmetric antiquark $\bar{\Psi}^{[kl]}$ by a string

$$B_\alpha = \int d\vec{z}\, Q^C_j(x_0) \left(P \exp i \int_{x_0}^{z} \vec{A}(y) d\vec{y}\right)^j_k \vec{\gamma}_{\alpha\beta}$$

$$\bar{\Psi}^{[kl]}_\beta(z) \left(P \exp i \int_{z}^{x_1} \vec{A}(y) d\vec{y}\right)^i_l Q_i(x_1)$$

By replacing the antisymmetric fermion by a gluino, we map the baryon of the orientifold theory to a fermion in $\mathcal{N} = 1$ SYM. The latter fermion in SYM is a superpartner of the meson.

Thus, by using planar equivalence, we can argue that up to $1/N$, heavy mesons of the form $\bar{Q}Q$ and baryons of the form QQq are mass degenerate.

The degeneracy improves as we neglect the end-points quarks. We thus predict the same Regge slope for asymptotically heavy mesons and baryons.

SUSY relics in pure Yang-Mills ?

We argue that one-flavor QCD can be approximated by a supersymmetric theory. In the old 't Hooft large-N expansion fundamental flavor is quenched and hence the one flavor QCD becomes at large-N pure Yang-Mills. So, perhaps there are remnants of supersymmetry in pure Yang-Mills ...

In supersymmetric theories

$$\langle \mathrm{tr}\, F^2 + i \mathrm{tr}\, F\tilde{F} \rangle = M_{\mathrm{UV}}^4 \exp - \tau/3$$

where τ is the holomorphic coupling.

We can estimate $F\tilde{F}$ from either Witten-Veneziano or lattice simulations. The value is

$$\frac{1}{8\pi^2} F^a \tilde{F}^a = 1.3 \times 10^{-2}\, \mathrm{GeV}^4$$

The gluon condensate in pure Yang-Mills was estimated (NSVZ, 1981)

$$\frac{1}{8\pi^2} F^a F^a = 1.2 \times 10^{-2}\, \mathrm{GeV}^4$$

Non-SUSY Seiberg duality

We can extend our proposal to include matter. In particular we proposed a SQCD like theory and non-SUSY Seiberg duality between an $U(N_c)$ electric theory and a $U(N_f - N_c + 4)$ magnetic theory.(Armoni, Israel, Moraitis and Niarchos, 2008)

OQCD-AS				
	$\mathrm{U}(N_c)$	$\mathrm{SU}(N_f)$	$\mathrm{SU}(N_f)$	$\mathrm{U}(1)_R$
A_μ	adjoint	$\bullet$	$\bullet$	0
λ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\bullet$	$\bullet$	1
$\tilde{\lambda}$	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\bullet$	$\bullet$	1
Φ	$\overline{\square}$	$\square$	$\bullet$	$\frac{N_f - N_c + 2}{N_f}$
Ψ	$\square$	$\square$	$\bullet$	$\frac{-N_c + 2}{N_f}$
$\tilde{\Phi}$	$\square$	$\bullet$	$\overline{\square}$	$\frac{N_f - N_c + 2}{N_f}$
$\tilde{\Psi}$	$\overline{\square}$	$\bullet$	$\overline{\square}$	$\frac{-N_c + 2}{N_f}$

Table 1: *The non-susy electric theory.*

OQCD-AS $(\tilde{N}_c = N_f - N_c + 4)$				
	$\mathrm{U}(\tilde{N}_c)$	$\mathrm{SU}(N_f)$	$\mathrm{SU}(N_f)$	$\mathrm{U}(1)_R$
A_μ	adjoint	$\bullet$	$\bullet$	0
λ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\bullet$	$\bullet$	1
$\tilde{\lambda}$	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\bullet$	$\bullet$	1
ϕ	$\square$	$\overline{\square}$	$\bullet$	$\frac{N_c-2}{N_f}$
ψ	$\overline{\square}$	$\overline{\square}$	$\bullet$	$\frac{N_c-N_f-2}{N_f}$
$\tilde{\phi}$	$\overline{\square}$	$\bullet$	$\square$	$\frac{N_c-2}{N_f}$
$\tilde{\psi}$	$\square$	$\bullet$	$\square$	$\frac{N_c-N_f-2}{N_f}$
M	$\bullet$	$\square$	$\overline{\square}$	$\frac{2N_f-2N_c+4}{N_f}$
χ	$\bullet$	$\square\square$	$\bullet$	$\frac{N_f-2N_c+4}{N_f}$
$\tilde{\chi}$	$\bullet$	$\bullet$	$\overline{\square\square}$	$\frac{N_f-2N_c+4}{N_f}$

Table 2: *The matter content of the proposed magnetic description of OQCD-AS, with number of colours* $\tilde{N}_c = N_f - N_c + 4$*.*

More applications ...

I haven't discussed many other applications of planar equivalence. In the past years planar equivalence was discussed in relation with

- Centre symmetry ($\mathcal{Z}_2$ or $\mathcal{Z}_N$?) and confinement. (Sannino, Shifman, Unsal, Del-Debbio, Patella)
- Non-supersymmetric type 0' string theory. (DiVecchia, Liccardo, Marotta, Pezzella)
- Baryons. (Bolognesi, Cherman, Cohen)
- Confinement/de-confinement transition. (Hollowood, Naqvi)
- Eguchi-Kawai reductions. (Kovtun, Unsal, Yaffe)
- The Hagedorn transition. (Cohen)
- M-theory membranes. (Naqvi)

Summary

I hope that I have convinced you that planar equivalence is a new and an exciting tool for studying the non-perturbative regime of QCD.

I would like to thank all my collaborators on this subject:

Sasha Gorsky, Emiliano Imeroni, Dan Israel, Biagio Lucini, Gregory Moraitis, Asad Naqvi, Vasilis Niarchos, Agostino Patella, Claudio Pica, Misha Shifman, Graham Shore, Mithat Unsal, Gabriele Veneziano.

And also to Thank you !