Rotating Strings
and
Spinning Glueballs

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based on

A.A., J. Barbon and A. Petkou, **"Orbiting Strings in AdS Black Holes and N=4 SYM at Finite Temperature"**, JHEP 06 (2002) 58.

A.A., J. Barbon and A. Petkou, **"Rotating Strings in Confining AdS/CFT Backgrounds"**, JHEP 10 (2002) 69.

Outline

- Introduction to the AdS/CFT correspondence
- Rotating strings and anomalous dimensions
- Orbiting strings in black-hole background and $\mathcal{N} = 4$ SYM at finite temperature
- Rotating strings in confining backgrounds
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Introduction

The AdS/CFT correspondence (Maldacena) is a powerful conjecture that relates $\mathcal{N} = 4$ SYM gauge theory to type IIB string theory on $AdS_5 \times S^5$ background.

The main ingredients of the duality dictionary are: $g_{st} \sim \frac{1}{N}$ and $\frac{R^2_{AdS}}{\alpha'} = \sqrt{\lambda} \equiv \sqrt{g^2_{YM} N}$.

For a theory with large number of color $N \to \infty$ and large 't Hooft coupling one can trust the supergravity approximation.

In other words: The strong coupling regime of the gauge theory is described in terms of classical (super-)gravity !

The conjecture was applied to calculations of Green function, Wilson loops, anomalous dimensions, etc. at strong coupling.

Moreover, a qualitative picture of confinement as well as chiral symmetry breaking was found.

An example:

suppose that we want to calculate the potential between a quark-antiquark pair, separated by a distance L, in $\mathcal{N} = 4$ SYM.

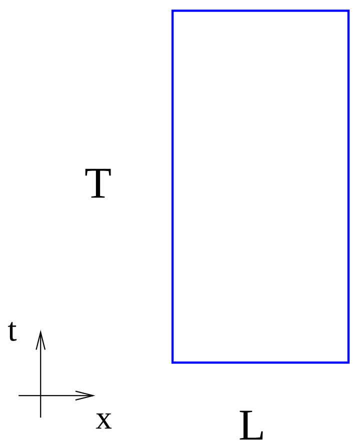
The perturbative result (tree-level) is a Coulomb potential

$$V_{\bar{q}q} = \frac{\lambda}{L}$$

At strong coupling the AdS/CFT prediction (Maldacena) is

$$V_{\bar{q}q} = \frac{\sqrt{\lambda}}{L}$$

The prescription, given by Polyakov and Maldacena is the following: The potential is related to a rectangular Wilson loop $< W >= \exp -TV(L)$, with L and T as shown in the figure

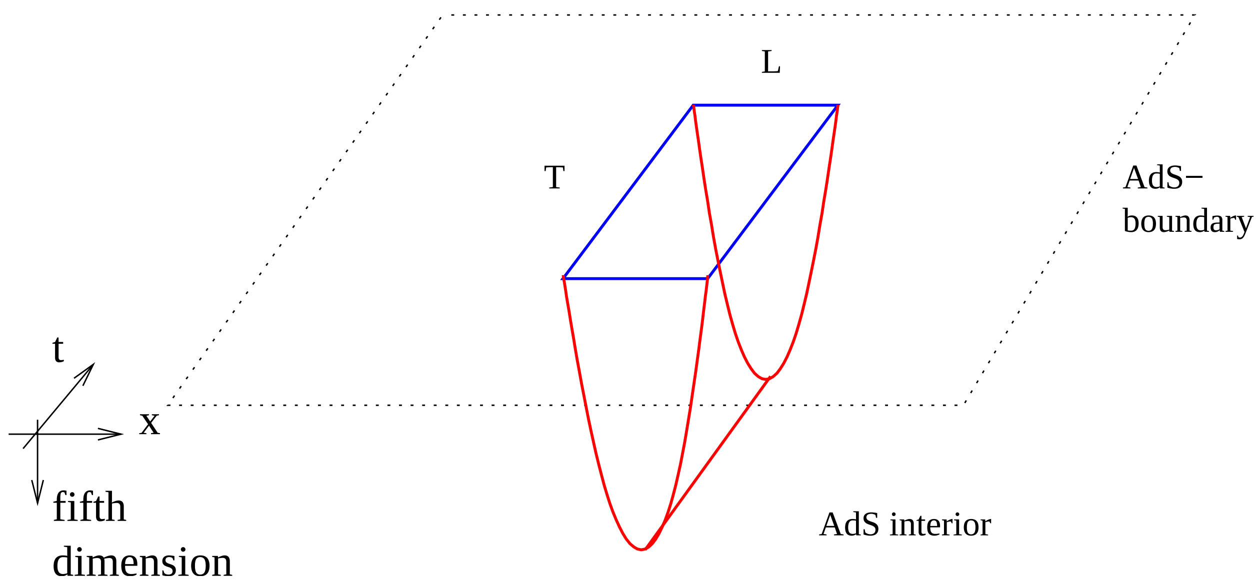


In order to calculate the quark-antiquark potential, according to the AdS/CFT prescription, one has to place the rectangular Wilson loop at the boundary of the AdS space, where the field theory 'lives'.

Then

$$< W >= \exp - S_{\mathrm{Nambu-Goto}}$$

The string worldsheet extends into the AdS bulk and the minimal surface (the string action) gives the quark-antiquark energy.



The proper length of the string is found by minimizing the Nambu-Goto action. It gives

$$\text{String} - \text{length} = \frac{R_{Ads}^2}{L},$$

and therefore

$$V(L) = \frac{\sqrt{\lambda}}{L}.$$

The purpose of this talk is to present a new prescription, due to Gubser, Klebanov and Polyakov, for calculating the energy of a "glueball" as a function of its spin.

The prescription is given for $\mathcal{N} = 4$ SYM and it is used to make a prediction about the dimension of twist two operator with large spin at strong coupling. The result is,

$$E - S = \Delta - S = \frac{\sqrt{\lambda}}{\pi} \log S$$

In contrast with the old perturbative result (Gross, Wilczek)

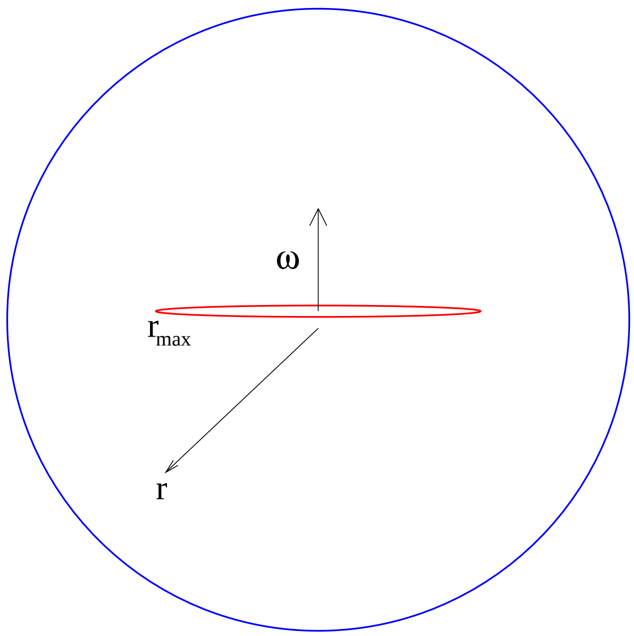
$$\Delta - S \sim \lambda \log S.$$

We will present finite temperature corrections to the above result. In addition, we will find a Regge trajectory for a confining gauge theory

$$E^2 = \sigma S$$

Rotating strings and anomalous dimensions

Gubser, Klebanov and Polyakov suggested to consider rotating string in AdS background



Due to the centrifugal force, the closed folded string will extend toward the boundary.

The idea is to calculate the energy and the spin of the string as a function of its angular velocity ω.

The Hamilton formulation of the AdS/CFT conjecture states that the Hilbert space of type IIB string theory on $AdS_5 \times S^5$ is the same as the the Hilbert space of $\mathcal{N} = 4$ SYM theory.

Therefore the rotating closed string should correspond to a "glueball" in the field theory. The simplest speculation is

$$|\Psi >= \mathrm{Tr}\ \Phi \nabla^S \Phi |0> .$$

Moreover, the energy and the spin of the closed string should correspond to the energy and the spin of the "glueball" $|\Psi>$.

Thus, by looking at the above simple classical configuration, one can study the behavior of glueballs in the gauge theory and to make a prediction about the strong coupling behavior.

The AdS metric in global coordinates takes the form

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega_3^2$$

with

$$\mathrm{d}\Omega_3^2 = \mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi_1^2 + \cos^2\theta\mathrm{d}\phi_2^2$$

and

$$f(r) = 1 + r^2.$$

Now let us plug the above metric in the Nambu-Goto action

$$I = -\frac{1}{2\pi\alpha'}\int \mathrm{d}\tau\mathrm{d}\sigma\sqrt{-\det G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu}\,.$$

It yields

$$I = -\frac{1}{2\pi\alpha'}\int \mathrm{d}\tau \int_0^{r_{max}} \mathrm{d}r\sqrt{1-\omega^2\frac{r^2}{1+r^2}}\,.$$

The energy and the spin take the form

$$E = \frac{\partial L}{\partial \omega}\omega - L = \frac{2}{2\pi\alpha'}\int_0^{r_{max}} \mathrm{d}r \frac{1}{\sqrt{1-\omega^2\frac{r^2}{1+r^2}}}$$

and

$$S = \frac{\partial L}{\partial \omega} = \frac{2}{2\pi\alpha'}\int_0^{r_{max}} \mathrm{d}r \frac{\omega\frac{r^2}{1+r^2}}{\sqrt{1-\omega^2\frac{r^2}{1+r^2}}}$$

The value of r_{max} is determined by the reality condition

$$r_{max}^2 = \frac{1}{\omega^2 - 1}.$$

For large r_{max} ($\omega \to 1$) we find

$$E = \frac{1}{\pi\alpha'}(r_{max}^2 + \log r_{max})$$

$$S = \frac{1}{\pi\alpha'}(r_{max}^2 - \log r_{max})$$

Such that

$$E - S = \frac{\sqrt{\lambda}}{\pi}\log S$$

Since time in global coordinates is conjugate to the scaling generator, the energy can be interpreted as the scaling dimension of the operator

$$\mathcal{O} = \mathrm{Tr}\ \Phi \nabla^S \Phi \,.$$

Classically,

$$\Delta = S + 2$$

(because the dimension of both Φ and ∇ is 1).

The strong coupling prediction for the anomalous dimension of $\mathcal{O}$ is

$$\gamma(\mathcal{O}) = \Delta - S = \frac{\sqrt{\lambda}}{\pi} \log S.$$

Orbiting strings in black-hole background and $\mathcal{N}=4$ SYM at finite temperature

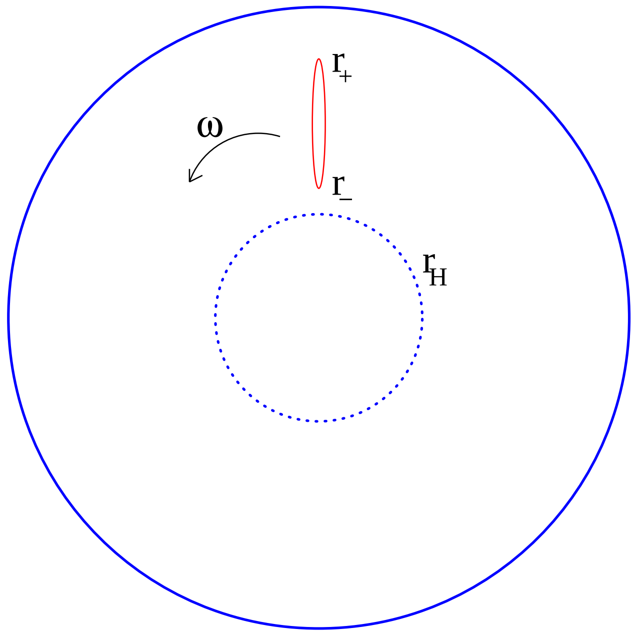
We would like now to find corrections to the GKP result, due to finite temperature effects. Namely, suppose that the glueball travels in a plasma of gluons. How the dispersion relation $E(S)$ is modified at strong coupling ?

In order to address this question, we will use Witten's conjecture that $\mathcal{N}=4$ SYM at finite temperature T is dual to type IIB string theory on AdS black-hole background. At large T the dictionary is $M_{bh} \sim T^4$.

The conjecture is obtained by looking at the near horizon geometry of near-extremal D3 branes (similarly to how Maldacena arrived to his conjecture).

So, we would like to ask how rotating strings extend in the background of a black-hole.

The solution is



The AdS black-hole metric is similar to the AdS metric, with the replacement

$$f(r) = 1 + r^2 \rightarrow f'(r) = 1 + r^2 - \frac{M}{r^2}$$

This modification leads to the following prediction for the dispersion relation

$$E - S = \frac{\sqrt{\lambda}}{2\pi} \log S/T^4$$

At large T. (At small T the black-hole is unstable due to Hawking radiation).

Is the orbiting string stable ?

There are several effects that make the orbiting string unstable.

First, if we turn on g_{st} the closed string will probably tear to smaller pieces of closed strings, since those configurations are energetically favorable.

In the field theory side it means that at finite N, the glueball will decay to lighter glueballs. Indeed, at finite N the Hilbert space is not expected to be described by a single trace states, but by a mixture of multi trace states ('t Hooft).

Suppose now that we are interested only in $g_{st} = 0$ namely in the extreme limit $N \to \infty$.

Is the configuration stable in this particular case ?

It turns out that the string rotates always with $\omega > 1$. It will transfer angular momentum to the black-hole that will start to rotate with an angular velocity Ω.

The thermodynamical equilibrium is achieved when the "chemical potentials"

$$\omega = \frac{\partial E}{\partial S}$$

will equate

$$\omega = \Omega.$$

However, black-holes are bounded by $\Omega < 1$.

It means that equilibrium cannot be achieved. The "planetoid" will transfer its energy to the black-hole and it will finally fall inside.

What is the picture of this phenomenon in the field theory side ?

It simply means that glueballs that travel in hot plasma are not stable.

The glueball will loose its energy to the plasma and finally it will melt to its gluonic composites.

Rotating strings in confining backgrounds

$\mathcal{N} = 4$ is a conformal theory and it does not exhibit confinement. It is, of course, interesting to ask what is the prediction of the new prescription for realistic confining models.

There is no known gravity solution in global coordinates that is dual to a confining gauge theory. We therefore consider the following toy model: A charged AdS black-hole.

The background is described by

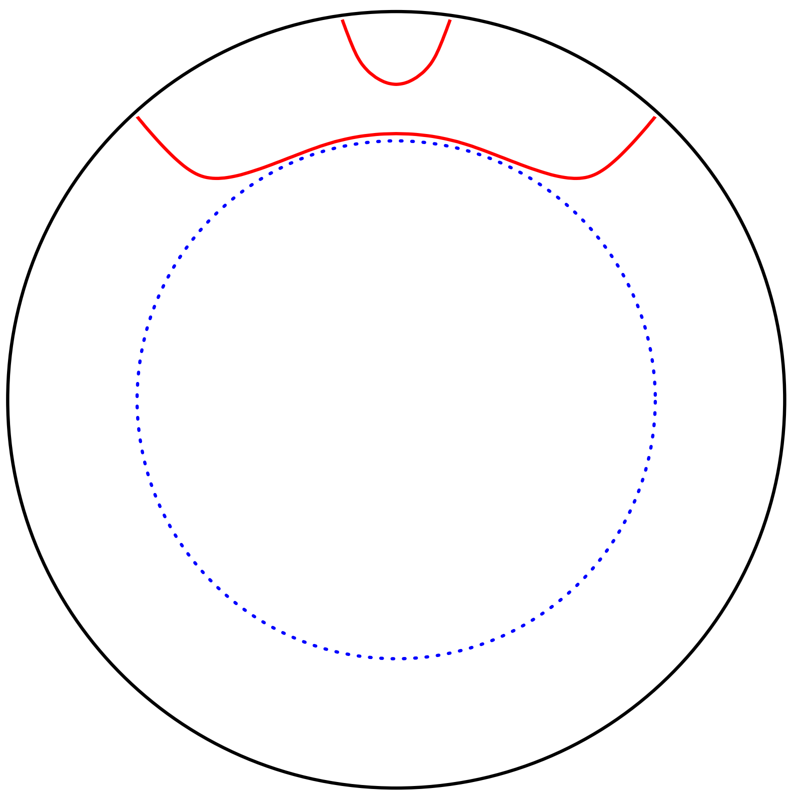
$$f(r) = 1 + r^2 \to f''(r) = 1 + r^2 - \frac{M}{r^2} + \frac{Q^2}{r^4}$$

We argue that in the limit $M \to 0, Q^2 \gg 1$ the background describes a confining gauge theory.

Though the background is unphysical, due to a naked singularity at $r = 0$, it will be a good toy model for studying confinement.

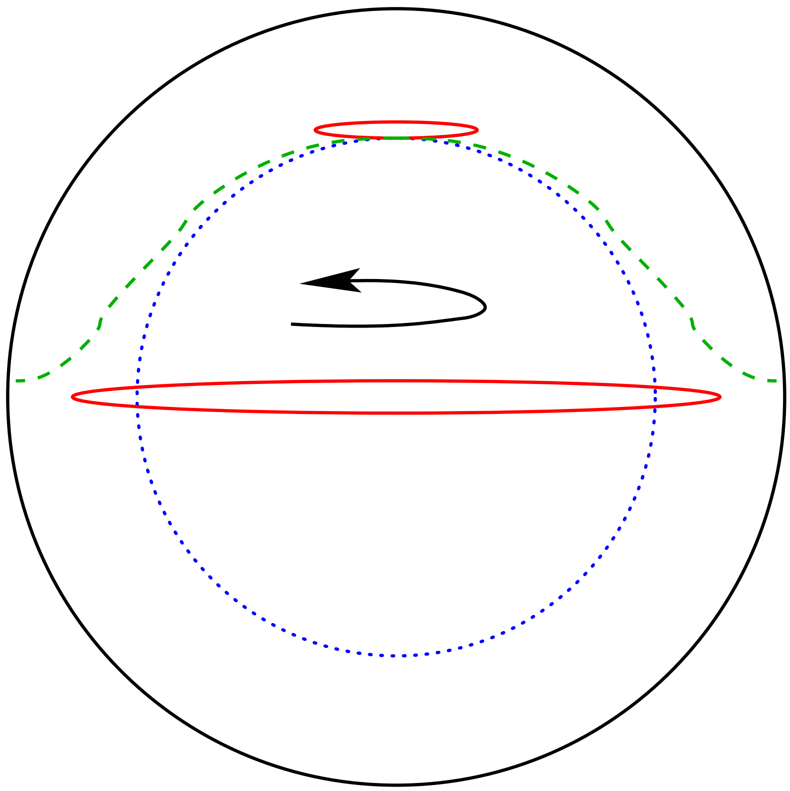
The reason is that $f''(r)$ has a minimum at $r_* \sim Q^{\frac{1}{3}}$. Backgrounds with a function $f(r)$ that has a minimum will lead to a confining potential $V(L) = \sigma L$ (Kinar, Schreiber and Sonnenschein).

For large separation L, the string will prefer to 'rest' on the minimum of $f(r)$



and therefore $V(L) = \mathrm{string} - \mathrm{length} \sim L$.

Let us place rotating strings in this background



The string that sits on the dotted blue line (the minimum of $f''(r)$) is the physical string. Upon increasing its angular momentum, it will extend and follow the dashed green line.

The 'central' string is not physical, but we will use it to study the physical string.

Short strings, $1 \ll S \ll \sqrt{\lambda} Q^{\frac{1}{3}}$ do not see the curvature of the space. They will exhibit a Regge trajectory

$$E^2 = \sigma S \,,$$

with $\sigma = \frac{Q^{\frac{2}{3}}}{\alpha'}$, the effective string tension that the rotating string sees. It is the same string tension that the Wilson loop string sees.

Thus, we have a prediction for the gauge theory glueballs: a Regge trajectory, similarly to the observed behavior of mesons.

If the spin (or energy) is large enough, $S \gg \sqrt{\lambda} Q$, the string will extend close to the boundary and it will feel the asymptotic AdS metric.

In this case we will get the GKP result that is related to asymptotic freedom

$$E - S = \frac{\sqrt{\lambda}}{\pi} \log S.$$

Finally, there is an intermediate regime, $\sqrt{\lambda}Q^{\frac{1}{3}} \ll S \ll \sqrt{\lambda}Q$, where

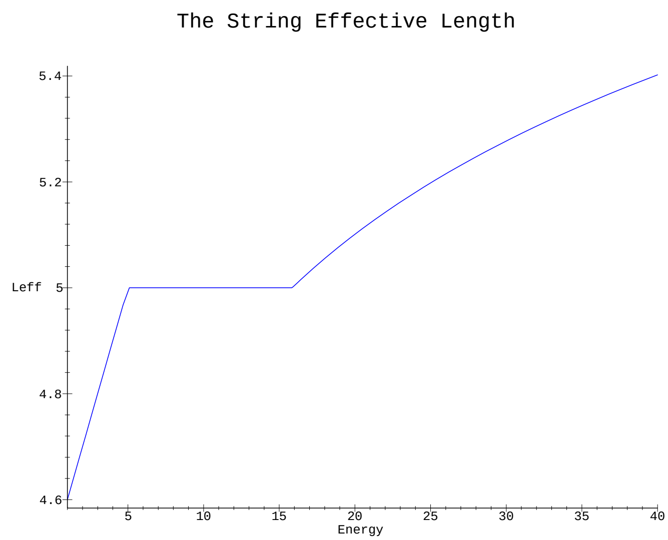
$$E - S = \sigma.$$

This is a new behavior which signals the appearance of new degrees of freedom - the glueballs (instead of free gluons in the UV).

It is interesting also to study a new physical quantity: the effective length of the string (the size of the glueball).

We define it as the worldsheet area per cycle.

The behavior of L_{eff} as a function of energy is



In between the linear and the logarithmic growth of the glueball there is an interesting plateau: The glueball does not grow as we increase its energy. We interpret this novel behavior due to a mixing of confinement and finite size effects.

Summary

The new prescription by Gubser, Klebanov and Polyakov is a useful tool to study gauge theories at strong coupling.

Their original result for the anomalous dimension of twist two operators

$$\Delta - S = \frac{\sqrt{\lambda}}{\pi} \log S$$

might have applications in deep inelastic scattering.

In our work, we have extended their result for a system at finite temperature.

In addition, we studied a toy model for confinement and we found a Regge trajectory for glueballs.

Finally, a new behavior for the size of glueballs as a function of the energy was found.