UV/IR mixing via Closed Strings and Tachyonic Instabilities

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Introduction

Non-commutative gauge theories attracted recently a lot of attention mainly due to the discovery of their relation to string/M theory (Connes, Douglas, Schwarz and Seiberg, Witten).

In particular is was found that a collection of N D3 branes with a constant NS-NS 2 form background leads in the Seiberg-Witten limit to a non-commutative $\mathcal{N} = 4$ SYM theory.

Non-commutative field theories are realized by replacing the ordinary multiplication by a $*$-product

$$f \star g(x) = e^{\frac{i}{2}\theta^{\mu\nu}\partial_\mu^{(\xi)}\partial_\nu^{(\eta)}} f(x+\xi)g(x+\eta)|_{\xi,\eta\to 0}.$$

In particular,

$$x^\mu \star x^\nu - x^\nu \star x^\mu = i\theta^{\mu\nu}.$$

The perturbative dynamics of these theories is interesting. In particular it was found (Minwalla, Van Raamsdonk, Seiberg) that certain non-planar diagrams might lead to a UV/IR mixing. Contributions from high momentum seem to affect the infra-red dynamics. This is a surprising phenomena, since usually the UV physics is irrelevant to the IR physics in the following sense

$$F \star F = F^2 + \theta \partial^2 F F + ...$$

In noncommutative theories it looks as if an infinite sum of irrelevant operators converge to a relevant one. The IR $F \star F$ theory seems to be different than the IR of the ordinary commutative Yang-Mills theory.

Another interesting aspect of the UV/IR mixing phenomena is that in non-supersymmetric gauge theories, the dispersion relation of the photon is modified as follows (Matusis, Susskind, Toumbas)

$$E^2 = \vec{p}^{\,2} - (N_B^{adj} - N_F^{adj})\frac{g^2}{\pi^2}\frac{1}{(\theta p)^2}$$

($N_{B,F}^{adj}$ are the numbers of bosons and fermions in the adjoint representation respectively).

In particular, pure non-commutative Yang-Mills theory is perturbatively tachyonic !

Is is not known at the moment whether this theory has a vacuum or not: the tachyonic instability might be due to an expansion around the wrong vacuum.

Our aim is to gain a better understanding of this phenomena by describing it via string theory.

We will see that the various non-planar diagrams which are responsible for the UV/IR can be described by a rather simple gauge invariant effective action.

This action can be derived also via string theory. In addition, string theory would provide an explanation for the UV/IR phenomena. We will see that the field theory tachyons can be related to closed strings tachyons (and all tower of massive scalars ! - as suggested by Arcioni, Barbon, Gomis and Vazquez-Mozo). In addition, milder UV/IR effects, which exist also in supersymmetric theories, can be related to closed strings massive 2-forms.

Non-Commutative Gauge Theories

We will be interested in non-commutative $U(N)$ Yang-Mills theory coupled to 6 scalars in the adjoint representation

$$S = \mathrm{tr} \int d^4x \left(-\frac{1}{2g^2} F_{\mu\nu} \star F^{\mu\nu} + D_\mu \phi^i \star D^\mu \phi^i \right)$$

with,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i(A_\mu \star A_\nu - A_\nu \star A_\mu)$$

and

$$D_\mu \phi^i = \partial_\mu \phi^i - i(A_\mu \star \phi^i - \phi^i \star A_\mu)$$

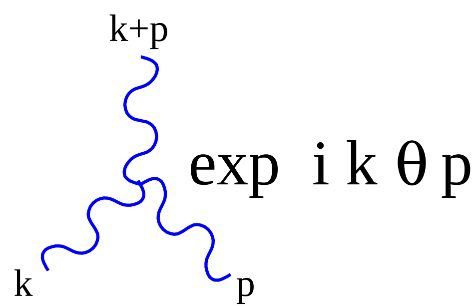
for $i = 1...6$. The model is invariant under non-commutative $U(N)$ gauge transformation

$$\delta_\lambda A_\mu = \partial_\mu \lambda - i(A_\mu \star \lambda - \lambda \star A_\mu)$$
$$\delta_\lambda \phi^i = -i(\phi^i \star \lambda - \lambda \star \phi^i).$$

Note that the $SU(N)$ theory is not gauge invariant (Matsubara). Note also that the $U(1)$ theory is not free (but, in fact, asymptotically free).

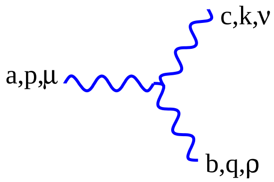
Perturbative analysis - UV/IR mixing

The Feynman rules of the non-commutative theory differ from the rules of the ordinary commutative theory by a phase which is added to the vertex



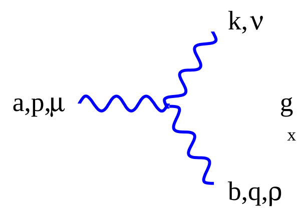
This modification has several interesting consequences:

- It turns out that planar graphs (at any order of perturbation theory) are the same as the planar graphs of the ordinary commutative theory, apart from global phases (Filk).
- One loop (and certain higher loop) non-planar graphs are UV finite.



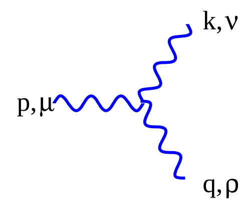
$g(\cos \tilde{p}q\, f^{abc} + \sin \tilde{p}q\, d^{abc})$
$\times\, (-p^{\rho} g^{\mu\nu} + \text{perm.})$

SU(N)-SU(N)-SU(N)



$g\, (2/N)^{1/2} \sin \tilde{p}q\;\; \delta^{ab}$
$\times\, (-p^{\rho} g^{\mu\nu} + \text{perm.})$

U(1)-SU(N)-SU(N)

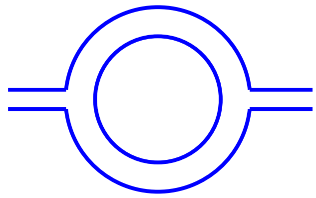


$g(2/N)^{1/2} \sin \tilde{p}q$
$\times\, (-p^{\rho} g^{\mu\nu} + \text{perm.})$

U(1)-U(1)-U(1)

Consider the vacuum polarization in non-commutative Yang-Mills theory. There are two contributions

(i). planar contribution

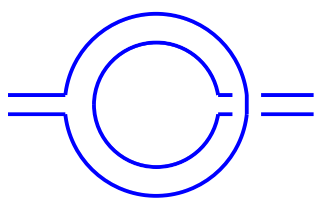


$\Pi_{\mu\nu}^{\text{planar}} = \Pi_{\mu\nu}(\text{commutative theory})$

It diverges logarithmically. Therefore it should be regularized and renormalized. Therefore

$\beta_0(\text{non} - \text{commutative } U(N) \text{ theory}) =$
$\beta_0(\text{commutative } SU(N) \text{ theory}).$

(ii). non-planar contribution



Non-vanishing only when the external legs are in the $U(1)$.

$\Pi_{\mu\nu}^{\rm non-planar} \sim$
$\int d^4k \; e^{2ik\theta p} \frac{1}{k^2} \frac{1}{(k+p)^2} (k_\mu k_\nu + ...) \sim$
$(\theta p)_\mu (\theta p)_\nu / (\theta p)^4 + (p^2 g_{\mu\nu} - p_\mu p_\nu) \log m^2 (\theta p)^2$

It is UV finite !

The pole and the log seem to be relevant in the IR, since they diverge at $p = 0$.

The source of these contributions is from high momenta. Thus the UV seems to affect the IR (Minwalla, Van Raamsdonk, Seiberg).

This is called "the UV/IR mixing" in non-commutative theories.

The coefficient in front of the pole in the general case is $N_B^{adj} - N_F^{adj}$. It vanishes for supersymmetric theories, but makes the pure NC Yang-Mills theory tachyonic.

The coefficient in front of the log is β_0. It leads to milder UV/IR mixing effects in supersymmetric theories (except $\mathcal{N} = 4$ SYM).

We will be interested also in the non-planar corrections to the 3 gluons vertex. For the case of 3 external gluons in the $U(1)$ the result is (with the notation $\tilde{p} = \theta p$)

$$\frac{\tilde{p}_1^\mu \tilde{p}_1^\nu \tilde{p}_1^\rho}{\tilde{p}_1^4} + \frac{\tilde{p}_2^\mu \tilde{p}_2^\nu \tilde{p}_2^\rho}{\tilde{p}_2^4} + \frac{\tilde{p}_3^\mu \tilde{p}_3^\nu \tilde{p}_3^\rho}{\tilde{p}_3^4} +$$
$$\sin(\tfrac{1}{2}\tilde{p}_1 p_2)(\log m^2 \tilde{p}_1^2 \; g^{\nu\rho} p_1^\mu + \text{perm.})$$

The first line is similar to the pole. Here we have milder $1/(\theta p)$ divergences. Again, these contributions vanish in the supersymmetric theory. In addition we have logarithms which survive the supersymmetric case.

We would like to list all the singular 2- and 3-point functions for the theory with 6 adjoint scalars.

Pole like contributions in the 2- and 3-point functions of the vectors and scalars

$$A^{\mu\nu}_{(1-1)} = \frac{64g^2N}{(4\pi)^2}\frac{\tilde{p}^\mu\tilde{p}^\nu}{\tilde{p}^4}.$$

$$A_{(1-1)} = \frac{32g^2N}{(4\pi)^2}\frac{1}{\tilde{p}^2}.$$

$$A^{\mu\nu\rho}_{(1-1-1)} = \frac{i64g^3\sqrt{N/2}}{(4\pi)^2}\left(\frac{\tilde{p}^\mu_1\tilde{p}^\nu_1\tilde{p}^\rho_1}{\tilde{p}^4_1} + \text{perm.}\right).$$

$$A^{\mu\nu\rho}_{(1-N-N)} = \frac{i64g^3\sqrt{N/2}}{(4\pi)^2}\ \frac{\tilde{p}^\mu_1\tilde{p}^\nu_1\tilde{p}^\rho_1}{\tilde{p}^4_1}.$$

$$A^{\mu}_{(1-1-1)} = \frac{i32g^3\sqrt{N/2}}{(4\pi)^2}\left(\frac{\tilde{p}^\mu_1}{\tilde{p}^2_1} + \text{perm.}\right).$$

$$A^{\mu}_{(1-N-N)} = \frac{i32g^3\sqrt{N/2}}{(4\pi)^2}\ \frac{\tilde{p}^\mu_1}{\tilde{p}^2_1}.$$

The information about these non-planar diagrams can be summarized in the following effective action

$$\frac{\pi^2}{2}S_I = g^2\int d^4p\ \Big(2\,\frac{\tilde{p}^\mu\tilde{p}^\nu}{\tilde{p}^4}\operatorname{tr}A_\mu(-p)\ \operatorname{tr}A_\nu(p) + \frac{1}{\tilde{p}^2}\operatorname{tr}\phi^i(-p)\ \operatorname{tr}\phi^i(p)\Big)$$
$$+\ \frac{i\,g^3}{(2\pi)^4}\int d^4p_1\,d^4p_2\,d^4p_3\,\delta(p_1+p_2+p_3)\times$$
$$\frac{\tilde{p}^\mu_1\tilde{p}^\nu_1\tilde{p}^\rho_1}{\tilde{p}^4_1}\ \operatorname{tr}A_\mu(p_1)\ \operatorname{tr}A_\nu(p_2)A_\rho(p_3)$$
$$+\frac{\tilde{p}^\mu_1}{\tilde{p}^2_1}\big(\operatorname{tr}A_\mu(p_1)\ \operatorname{tr}\phi^i(p_2)\phi^i(p_3) + 2\operatorname{tr}\phi^i(p_1)\ \operatorname{tr}\phi^i(p_2)A^\mu(p_3)\big).$$

Log like contributions in the 2- and 3-point functions of the vectors and scalars

$M^{\mu\nu}_{(1-1)} = -\frac{26g^2N}{3(4\pi)^2}(p^2 g^{\mu\nu} - p^\mu p^\nu)\log m^2\tilde{p}^2.$
$M_{(1-1)} = -\frac{26g^2N}{3(4\pi)^2}p^2\log m^2\tilde{p}^2.$
$M^{\mu\nu\rho}_{(1-1-1)} =$
$-\frac{i26g^3\sqrt{N/2}}{3(4\pi)^2}\sin(\frac{1}{2}\tilde{p}_1 p_2)(\log m^2\tilde{p}_1^2\ g^{\nu\rho}p_1^\mu +$
perm.).
$+ \ldots$

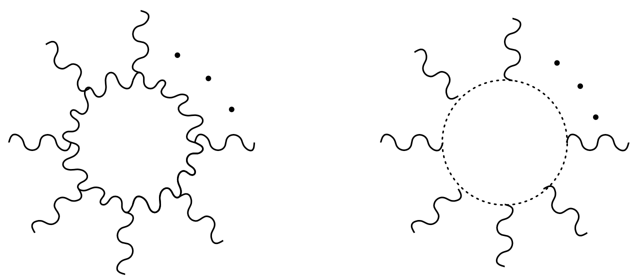
These log-like amplitudes can be summarized by the following effective action

$-\frac{24\pi^2}{13}S_{II} = g^2\int d^4p\ ((p^2 g^{\mu\nu} - p^\mu p^\nu)$
$\log m^2\tilde{p}^2(\mathrm{tr}\ A^\mu(-p))(\mathrm{tr}\ A^\nu(p)) +$
$p^2\log m^2\tilde{p}^2(\mathrm{tr}\ \phi^i(-p))(\mathrm{tr}\ \phi^i(p) +$
$\frac{ig^3}{(2\pi)^4}\int d^4p_1\ d^4p_2\ d^4p_3\ \delta(p_1+p_2+p_3)\times$
$\sin(\frac{1}{2}\tilde{p}_1 p_2)\times\log m^2\tilde{p}_1^2\ p_1^\mu\times$
$(\mathrm{tr}\ A^\nu(p_1))(\mathrm{tr}\ A^\mu(p_2)A^\nu(p_3)) +$
$(\mathrm{tr}\ \phi^i(p_1))(\mathrm{tr}\ A^\mu(p_2)\phi^i(p_3)).$

The effective action - Field theory derivation

We would like to find a gauge invariant effective action which reproduces the pole like terms.

Focusing on the simple case of non-commutative pure $U(1)$ theory, the relevant diagrams are



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The calculation yields

$$S^{I}_{eff} = \frac{1}{2\pi^2}\sum_{N=2}^{\infty}(ig)^N \int d^4p \sum_{n=1}^{N-1} \frac{(-)^n}{n!(N-n)!}\frac{m^2}{\tilde{p}^2}K_2(m\tilde{p}) \tilde{p}^{\mu_1}...\tilde{p}^{\mu_N}[A_{\mu_1}..A_{\mu_n}]_{\star n}(-p)[A_{\mu_{n+1}}..A_{\mu_N}]_{* N-n}(p)$$

The pole like (in fact, a Bessel function for $m > 0$) contributions can be written in the following compact form

$$S^{I}_{eff} \sim \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\ W(p)\ \mathrm{tr}\ W(-p) \frac{m^2}{\tilde{p}^2} K_2(m\tilde{p})$$

With $W(p)$ an open Wilson line, defined as follows

$$W(p) = \int d^4x\ P_\star \left(e^{i\,g \int_0^1 d\sigma\, \tilde{p}^\mu\, A_\mu(x+\tilde{p}\sigma)} \right) \star e^{ipx}$$

It suggests that the fundamental objects that are involved in the dynamics are open Wilson lines (dipoles). As we shall see, the action W propagator W has a simple explanation in terms of closed strings.

Similarly to the poles, the logarithms in the effective action can be reproduced from

$$S^{II}_{eff} \sim \beta_0 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\, F_{\mu\nu} W\,(p)\ \mathrm{tr}\, F_{\mu\nu} W\,(-p) K_0(m\tilde{p})$$

Type 0 String Theory

In order to relate our field theory results to strings, we need a non supersymmetric string theory - the type 0B string theory.

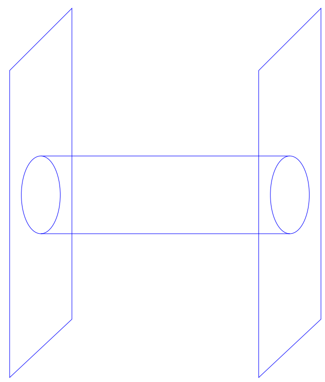
The type 0B string is obtained by a non-chiral GSO projection which leads to a purely bosonic closed string sector. It consists of the universal NS-NS sector with the same massless fields as in type IIB G_{MN}, B_{MN}, Φ, a Tachyon T and a doubled set of R-R fields (compared to the type IIB string). The doubling of the R-R fields leads to two kinds of D-branes: 'electric' and 'magnetic'. We will be interested in the theory which lives on the 'electric' D3 branes.

In order to have non-commutativity we will add a constant B_{MN} background field in the $1, 2$ directions.

The resulting field theory on the brane is obtained by dimensional reduction of the pure non-commutative 10d Yang-Mills theory to 4d.

The effective action - String Theory derivation

We will use the open/close string duality of the annulus



It can be viewed either as a closed string exchange or as an open string that circulates in the loop.

Consider an exchange of a closed string tachyon. In the ordinary theory it couples to the brane tension (Klebanov,Tseytlin)

$$\frac{N}{4(2\pi\alpha')^2}$$

and therefore at leading order in α' the tachyon does not couple to gauge fields on the brane. It renormalizes the vacuum energy.

In the non-commutative theory, the tachyon coupling to the brane involves an open Wilson line (Okawa, Ooguri)

$$S = \frac{\kappa_{10}}{g_{YM}^2} \int \frac{d^{10}P}{(2\pi)^{10}} \sqrt{\det \mathrm{G}}\, T(P)\, \mathcal{O}(-P)\,,$$

with

$$\mathcal{O}(P) = \frac{1}{4\,(2\pi\alpha')^2}\, \mathrm{tr} \int d^4x\, W(x,C) \star e^{ipx}\,.$$

We denote by P_M the 10-dimensional momentum, p_μ the momentum along the 4-dimensional world-volume of the D3-brane and $p_{\perp i}$ the momentum in the transverse directions. $W(x,C)$ now involves also the transverse scalars

$$W(x,C) = P_\star \left(e^{i\,g \int_0^1 d\sigma\, \tilde{p}^\mu A_\mu(x+\tilde{p}\sigma) + y_i \phi^i(x+\tilde{p}\sigma)} \right)$$

where we have defined $y_i = 2\pi\alpha' p_{\perp i}$ and $\phi^i = X^i/2\pi\alpha'$ $(i = 1, ..., 6)$.

In general, a closed string exchange between two D3 branes separated by a distance r, would lead to the following effective action

$$S^I_{eff} = \frac{\kappa_{10}^2}{g_{YM}^4} \int \frac{d^{10}P}{(2\pi)^{10}} \frac{\det G}{\sqrt{\det g}} \mathcal{O}(P)\mathcal{O}(-P) \frac{e^{ip_\perp r}}{\frac{M^2}{(2\pi\alpha')^2} + p_\perp^2}$$

M^2 is the effective mass of the tachyon propagating in the six transverse directions. $M^2 = \tilde{p}^2 - 8\pi^2\alpha'$. It is obtained by writing $P_M g^{MN} P_N = -\frac{2}{\alpha'}$ and expressing the closed string metric in terms of the open string metric by using the Seiberg-Witten relation $g^{-1} = G^{-1} - \theta G\theta/(2\pi\alpha')$. Note that the bare mass of the tachyon is subleading in comparison to the non-commutative momentum $\tilde{p}$. Thus, in principle, each scalar in the closed string tower which couples to the brane tension will contribute.

The four dimensional action takes the form

$$S^I_{eff} = \frac{\pi}{(4\pi\alpha')^4} \int \frac{d^4p}{(2\pi)^4} \sqrt{\det G}\, \mathrm{tr}W(p)\, \mathrm{tr}W(-p)\, G(p)$$

where

$$G(p) = \int \frac{d^6y}{(2\pi)^6} \frac{e^{iym}}{M^2 + y^2}\,.$$

Taking the field theory limit $\alpha' \to 0$, while keeping only the $O(\alpha'^4)$ term in the propagator (hidden in M^2) gives

$$G(p)|_{\alpha'^4} = \frac{4\pi^5}{3} \frac{m^2}{\tilde{p}^2} K_2(m\tilde{p})$$

which agrees with the field theory computation up to an overall factor !

The difference in the overall factor is due to exchange of massive closed strings that couple to the brane tension.

In the superstring case the NS-NS tower and the R-R tower will contribute with opposite signs, yielding a zero in front of the action.

The logarithmic action can also be derived by closed strings exchange.

Repeating the same steps, but now with a closed string 2-form, that couple to

$$\mathcal{O}^{\mu\nu}(P) = \frac{1}{2\pi\alpha'}\mathrm{tr}\int d^4x\, L_*(F^{\mu\nu}W(x,C)) * e^{ipx}$$

$$\mathcal{O}^{\mu i}(P) = \frac{1}{2\pi\alpha'}\mathrm{tr}\int d^4x\, L_*(D^{\mu}\phi^i W(x,C)) * e^{ipx}$$

and keeping the α'^2 piece in the propagator we find

$$G(p)|_{\alpha'^2} \sim K_0(m\tilde{p})$$

which yields the effective action for the supersymmetic field theory.

The overall factor, which is β_0, involves the understanding of the coupling of all the tower to the field strength F_{MN}.

Discussion

The effective action of the non-commutative field theory can be derived from string theory.

The tachyonic part of the action is related to an exchange of a closed string tachyon, in contrast to the part which survives in the supersymmetric case, which is related to massive closed strings exchange.

An attempt to remove the tachyon by a special orientifold (Sagnotti) will result with a type I like theory without a B-field and no realization of non-commutativity.

In addition in all cases of string theories with a bulk tachyon more bosons than fermions on the brane. For example: field theories that live on D3 branes (of type IIB/0B) placed on R^6/Γ orbifold singularity.

Therefore, though all the tower of scalars contributes to the tachyonic effective action, we suggest an (indirect) relation between the presence of the bulk tachyon and the tachyonic instabilities on the brane.

There might be a relation between closed string tachyon condensation in type 0 string theory and the consistency of the tachyonic non-commutative field theories.

It was conjectured (Bergman and Gaberdiel, Costa and Gutperle, and more recently by David, Gutperle, Headrick and Minwalla) that the true vacuum of the type 0 string is the supersymmetric type II string !

Thus, the tachyonic instabilities of the noncommutative theory might be an artifact of an expansion around the wrong (perturbative) vacuum.