Planar Equivalence: From Type 0 Strings to QCD Phenomenology

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Background (Orbifold field theories)

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Introduction

The study of non-supersymmetric gauge theories at strong coupling is of great importance. However, it is difficult to derive exact results for non-supersymmetric theories.

In supersymmetric theories the situation is better due to holomorphicity.

Strassler conjectured that a certain part of the non-perturbative physics, can be copied from a supersymmetric parent theory to a non-supersymmetric large N "orbifold field theory" daughter.

The purpose of this work is to present a concrete non-supersymmetric example, an "orientifold field theory", where some *exact* results can be obtained.

In addition, we will be able to derive, analytic non-perturbative results in QCD !

The model is an $SU(N)$ gauge theory with a Dirac fermion in the two-index antisymmetric representation, $\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$. It is very similar, at large N, to pure $\mathcal{N} = 1$ Super Yang-Mills theory.

The main results, restricted to the large N theory, are:

1. The model has a zero cosmological constant.

2. The hadrons are purely bosonic, even/odd parity degenerate, and with masses values exactly as in the supersymmetric theory.

3. The model has N degenerate vacua.

4. An exact bi-fermion condensate, $\langle \bar{\Psi}_L \Psi_R \rangle = N\Lambda^3 \exp i\frac{2\pi k}{N}, \quad k = 0 \ldots N-1$

5. There are “BPS” domain walls.

6. Certain correlation function are exactly as in $\mathcal{N} = 1$ SYM.

7. NSVZ β-function.

8. More ...

Though the model is very similar to $\mathcal{N} = 1$ SYM and one might suspect that at $N \to \infty$ it becomes $\mathcal{N} = 1$ SYM, it is not so.

The hadronic spectrum of the orientifold theory is purely bosonic. One cannot form a light colour-singlet fermion by using quarks in the anti-symmetric representation.

On the other hand the hadronic spectrum of $\mathcal{N} = 1$ SYM is bose-fermi degenerate.

Plan of the talk

- Introduction + Main Results
- String theory setup
- Perturbative proof of the statement
- Non-Perturbative arguments
- Results
- The Orientifold large-N expansion
- Applications to one-flavor QCD
- Applications to three-flavors QCD
- Remarks on gauge/gravity correspondence
- Conclusions

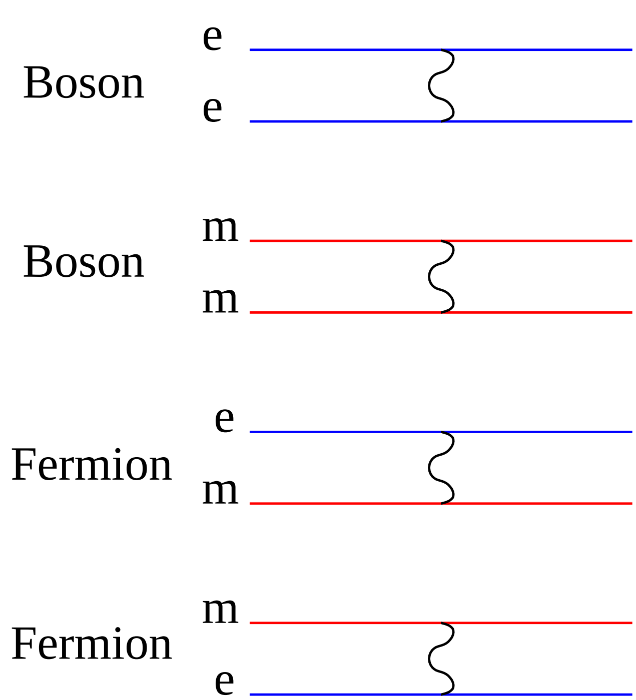
String theory setup

The "orientifold field theory" has a nice embedding in type 0A string theory. This bosonic closed string theory (with worldsheet supersymmetry) contains the following low-energy modes

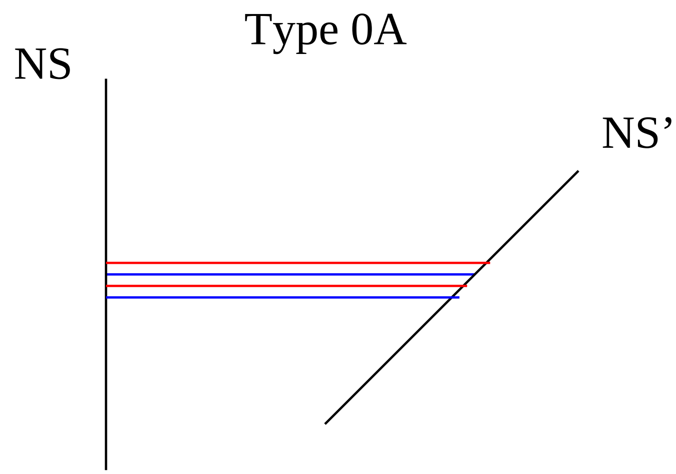
NS-NS: $\Phi, G_{\mu\nu}, B_{\mu\nu}, T$

R-R: $2 \times\ C, C_{\mu\nu}, C_{\mu\nu\rho\lambda}$ (in type 0B)

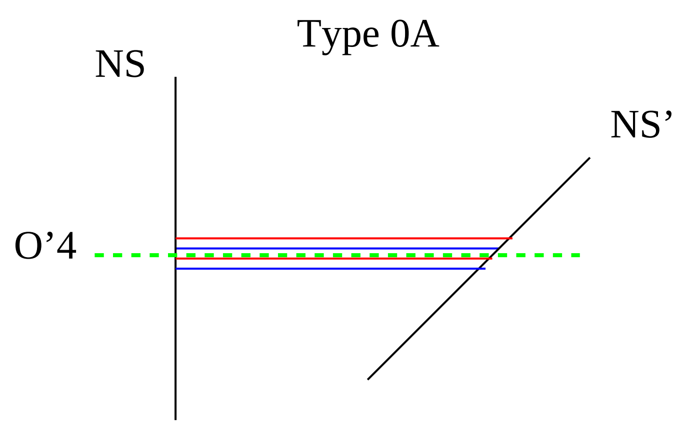
Accordingly there are two types of D-branes, 'electric' and 'magnetic'



The following brane configuration realises a $U_{\mathrm{e}}(N) \times U_{\mathrm{m}}(N)$ with bi-fundamental fermions



If we add a special orientifold plane $\Omega' = \Omega(-1)^{f_R}$ (Sagnotti) we get the "orientifold field theory"



The orientifold identifies the two R-R forms — hence leads to a single $U(N)$ factor.

The closed string spectrum of Sagnotti's model is simply the bosonic (NS-NS and R-R) truncation of the type II string (the closed string tachyon of the type 0 string is projected out). This is in agreement with field theory: the glueball spectrum of the 'orientifold field theory' at large-N is the bosonic truncation of its supersymmetric parent.

Moreover, if we think about the closed string theory as the MASTER-FIELD (a classical theory that controls the large-N gauge theory), then we expect that at large-N all bosonic Green functions, including

$\langle \mathrm{tr}\, F^2(x_1) \mathrm{tr}\, F^2(x_2) ... \mathrm{tr}\, F^2(x_n) \rangle$

coincide.

Perturbative proof

At the perturbative level the planar "orientifold field theory" is equivalent to planar $\mathcal{N} = 1$ SYM.

The planar equivalence follows from the Feynman rules of the two theories

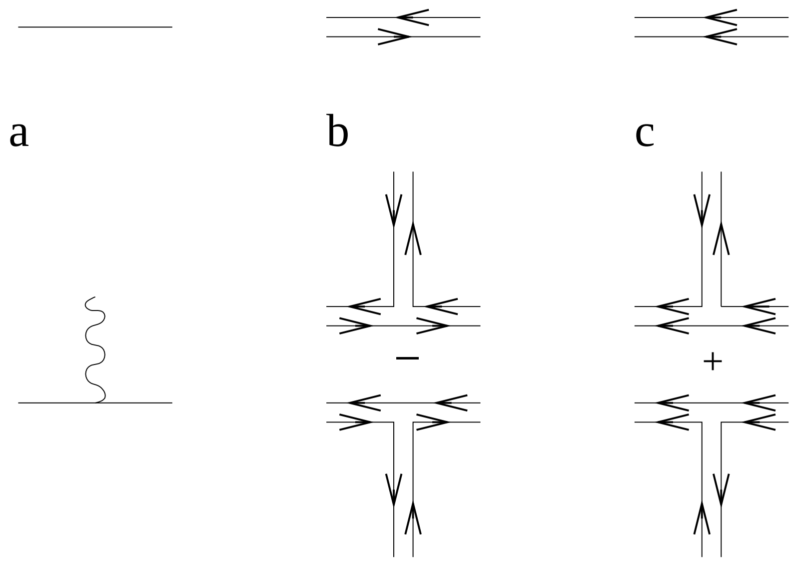


Figure 1: a. The fermion propagator and the fermion-gluon vertex. b. For $\mathcal{N} = 1$ SYM. c. For the orientifold theory.

An example: a planar contribution to the vacuum polarisation.

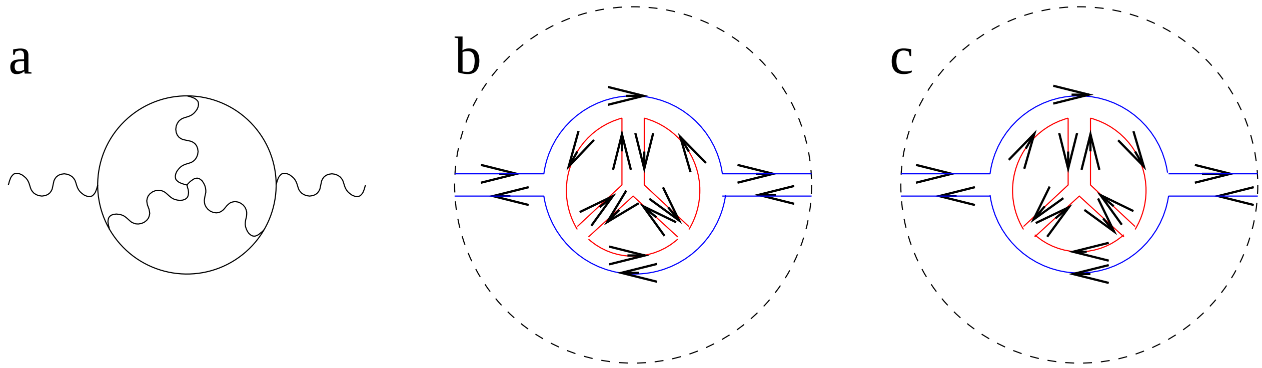


Figure b. is for $\mathcal{N} = 1$ SYM. Figure c. is for the orientifold theory.

In order to pass from b. to c., one has to reverse the orientation of the red lines.

For planar graphs (drawn on the sphere) the red lines and the blue lines do not intersect. It is as if we had two types of gluons: a red gluon and a blue gluon ('magnetic' and 'electric' gluons in the type 0 string theory description). This is why the two theories are perturbatively equivalent.

Non-Perturbative proof

Since the orientifold theory and $\mathcal{N} = 1$ SYM theory are similar to each other, but differ in their fermion field representation, we wish to compare their partition function after an integration over the fermion field.

We claim that after integration the two partition functions coincide at large N.

An intuitive argument: since the Feynman graphs of the two theories agree at large N, the Casimirs of the adjoint representation and the anti-symmetric representation should coincide at large N. But the partition function is a number that depends on Casimirs. At large N we should obtain the same number.

Let us present a more formal proof:

The partition function of $\mathcal{N}=1$ SYM is

$$\mathcal{Z}_0 = \int \mathcal{D}A \, \exp\left(iS[A]\right) \det\left(\not{\partial} + \not{A}^a T^a_{\text{adj}}\right)$$

Next, we will make use of the fact that

$$T^a_{\text{adj}} \sim T^a_{\Box \times \overline{\Box}} = T^a_{\Box} \otimes 1 + 1 \otimes T^a_{\overline{\Box}},$$

to write the partition function as follows

$$\mathcal{Z}_0 = \int \mathcal{D}A \, \mathcal{D}B \, \delta(A-B) \, \exp -\tfrac{1}{2}(S[A] + S[B])$$
$$\det\left(\not{\partial} + \not{A}^a (T^a_{\Box} \otimes 1) + \not{B}^a (1 \otimes T^a_{\overline{\Box}})\right)$$

The determinant is gauge invariant and hence it can be written formally by using Wilson loops (the detailed expansion can be found in ASV, hep-th/0412203, see also A. Patella, hep-lat/0511037.)

$\mathcal{Z}_0 = \int \mathcal{D}A\, \mathcal{D}B\, \delta(A - B) \exp -\frac{1}{2}(S[A] + S[B])$
$\sum \prod \mathcal{W}(A)\mathcal{W}(B)$.

Or $\mathcal{Z}_0 = \sum \langle \prod \mathcal{W}\mathcal{W} \rangle$

The partition function of the "orientifold field theory" is similar, but with reversed orientation for the red Wilson loop $\mathcal{W}^\star$.

At large-N gauge invariant amplitudes factorise. For example

$\langle \mathcal{W}(A)\mathcal{W}(B) \rangle = \langle \mathcal{W}(A)\mathcal{W}^\star(B) \rangle = \langle \mathcal{W} \rangle \langle \mathcal{W} \rangle$

(since reversing the orientation is as replacing a quark by an anti-quark and it does not change the expectation value of the Wilson loop).

More generally, we proved that at large-N
$\langle \prod \mathcal{W}_{\rm adj.} \rangle_{\rm conn.} = \langle \prod \mathcal{W}_{\rm anti-symm.} \rangle_{\rm conn.}$

So, the partition functions of the two theories coincide.

Results

Due to the relation with $\mathcal{N}=1$ SYM we arrive to the following results:

1. A zero cosmological constant. It follows from the relation $-i\rho V = \log \mathcal{Z}$ and the equality of the two partition functions.

2. The same bosonic IR degrees of freedom. The reason is that masses correspond to poles in amplitudes. One can add a source J to the gauge field. The partitions functions in the presence of the source $\mathcal{Z}_{J(x)}$ are also the same. Then, all bosonic amplitudes should be the same in the two theories. The spectrum of the "orientifold field theory", however, is purely bosonic.

3. We can couple a source $J(x)$ to $\bar{\Psi}_L \Psi_R(x)$ and integrate over the fermions to find

$$\langle \bar{\Psi}_L \Psi_R(x_1) \bar{\Psi}_L \Psi_R(x_2) ... \bar{\Psi}_L \Psi_R(x_n) \rangle = \text{const.}$$

4. There will be a fermion bi-linear condensate $\langle \bar{\Psi}_L \Psi_R \rangle = N\Lambda^3 \exp i \frac{2\pi k}{N}$ with N degenerate vacua.

5. Since the vacua are degenerate there will be domain-walls. Their exact tension is given, as $\mathcal{N}=1$ SYM, by the difference in the value of the bi-fermion condensate.

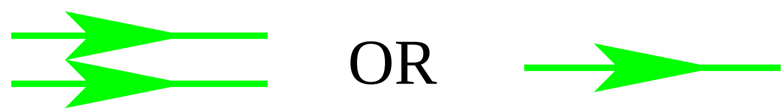
In conclusion, almost every result that we know about the bosonic part or about fermion bi-linears in $\mathcal{N}=1$ SYM theory can be copied to the "orientifold field theory".

On the other hand we cannot say anything about quantities that involve $(\bar{\Psi}\Psi)^n, \; n>1$. Probably the two theories are different in this sector.

The Orientifold Large-N Expansion

The results that were obtained so far for the large-N theory can be applied to QCD.

For $SU(3)$ with N_f quarks in the fundamental representation, we have the option of thinking about the quarks as if they transform in the antisymmetric representation



Let us choose the latter option. The idea is then to generalise to $SU(N)$ with N_f flavors in the antisymmetric representation. Finally, one can take the large-N limit, while keeping N_f as well as the 't Hooft coupling fixed.

This is a new kind a large-N expansion where quarks loops count like gluons loops.

Applications to one-flavor QCD

For the specific case of 1-flavor QCD, the large N theory becomes supersymmetric (in the bosonic sector).

One, thus, can copy results from $\mathcal{N}=1$ SYM to 1-flavor QCD, with an expected error of $1/N = 1/3$.

We expect to have a degeneracy between the odd parity hadrons and the even parity hadrons.

In particular, for the η' and the σ (the lowest 'glueballs' in the tower) we expect

$$M^2_{\eta'}/M^2_{\sigma} = 1 + O(1/N)$$

(actually, in this specific case, $1/N$ corrections are expected to be large).

The quark condensate in 1-flavor QCD

An important application of our result is the calculation of the quark condensate in 1-flavor QCD. We can simply copy the value from SUSY Yang-Mills.

For the SUSY theory the value of the gluino condensate is $\langle\lambda\lambda\rangle = -6N\Lambda^3$

The $1/N$ corrections are expected to be large in the "orientifold" theory. An important hint is that for $SU(2)$ the condensate should vanish, since the antisymmetric representation is equivalent to the singlet. Therefore we expect $\langle\bar{\Psi}_L\Psi_R\rangle \sim -6(N-2)\Lambda^3$

Thus we find
$\langle\bar{\Psi}_L\Psi_R\rangle_{2\ \mathrm{GeV}} = -(0.6\ \text{ to }\ 1.1)\Lambda^3_{\bar{\mathrm{MS}}}$

In a nice agreement with lattice results
$\langle\bar{\Psi}_L\Psi_R\rangle_{2\ \mathrm{GeV}} = -(0.4\ \text{ to }\ 0.9)\Lambda^3_{\bar{\mathrm{MS}}}$

Applications to three-flavors QCD

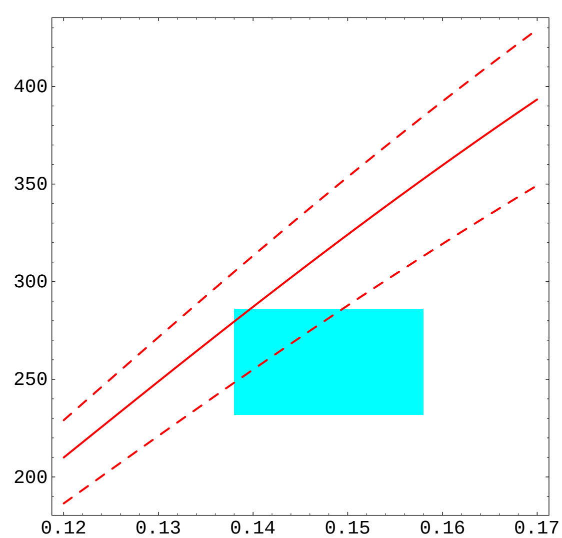
Recently (with G. Shore and G. Veneziano) we generalised our analysis to three-flavors QCD !

The idea is to consider an $SU(N)$ theory with one-flavor in the antisymmetric representation and two additional flavors in the fundamental representation.

This model reduces to three flavors QCD for $SU(3)$, becomes SUSY in the bosonic sector in the large N limit, but differs from the previous model by the $1/N$ corrections.

Our result for the QCD quark condensate is

$$\langle \bar{\Psi}\Psi \rangle_{2\,\mathrm{GeV}} \quad = \quad -\,(317\ \pm\ 30\ \pm\ 36\ \mathrm{MeV})^3$$



Gauge/Gravity correspondence

A great challenge is to estimate the $1/N$ corrections. From field theory the only hint about $1/N$ corrections is that for $N = 2$ the antisymmetric representation becomes a singlet, so certain quantities, such as the quark condensate vanish.

String theory (the Gauge/Gravity correspondence) might be useful as well. The 'orientifold field theory' has a realisation in type 0'B string theory on a stack of wrapped D5-branes.

In such setups the total RR flux is $N - 2$ due to the presence of an orientifold 5-plane. Certain quantities, which are 'sensitive' to the background RR flux, such as the chiral anomaly, are shifted $N \to N - 2$.

With E. Imeroni, we used it to make a couple of predictions about the finite-N theory. A concrete example is

$$M_{\eta'}/M_\sigma \sim C_-/C_+ \sim (N-2)/N$$

Conclusions

Exact non-perturbative results can be copied from the supersymmetric theory to the non-supersymmetric "orientifold field theory" (at large N).

Other aspects that I haven't discuss in detail are:

- Generalisations: $\mathcal{N} = 1$ with matter $\mathcal{N} = 2$ and $\mathcal{N} = 4$.
- The AdS/CFT correspondence.
- Further applications for QCD.