Everything You Wanted To Know About k-Strings But You Were Afraid To Ask

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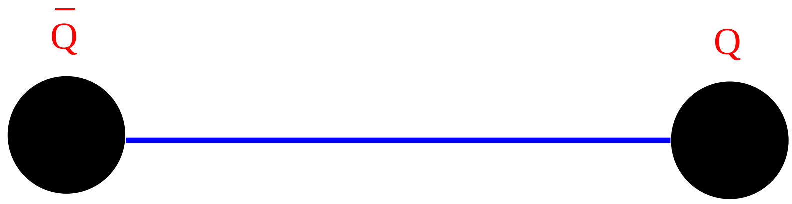
Introduction

In $SU(N)$ gauge theories with adjoint matter (or no matter at all), consider a large Wilson loop in a representation R,

$$W_R = \mathrm{tr} \exp i \int d\tau (A^a_\mu T^a_R \dot{x}^\mu)$$

A large rectangular Wilson loop corresponds to a well-separated quark anti-quark pair in a representation R.



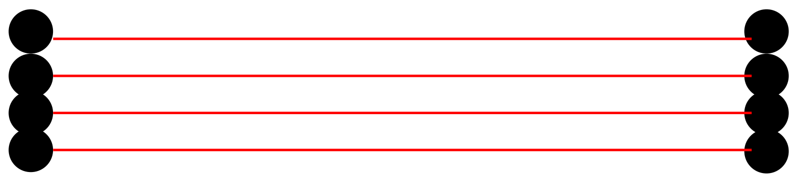
In $d > 2$ we expect

$$\langle W_R \rangle = \exp -\sigma_k \mathcal{A}$$

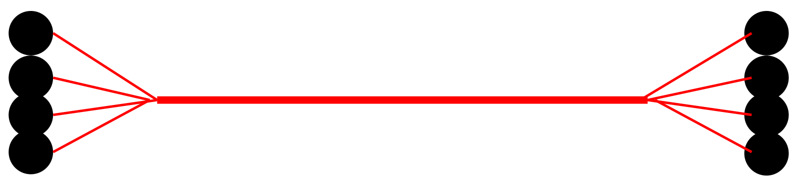
Or equivalently $V(L) = \sigma_k L$. The QCD-string that connects the quark anti-quark pair is called a k-string. The tension σ_k is expected to be a function of the N-ality of the quark. The N-ality is the charge of the quark field under the center of the gauge group $\exp i2\pi k/N$.

An intuitive way of thinking about the k-string is as a bound state of k elementary strings that connect a quark anti-quark pair in the fundamental representation.

If we bring k-elementary strings close to each other



they will attract each other and form a bound state



So, obviously we assume that $\sigma_k < k\sigma_1$. The difference $\sigma_k - k\sigma_1$ is the binding energy.

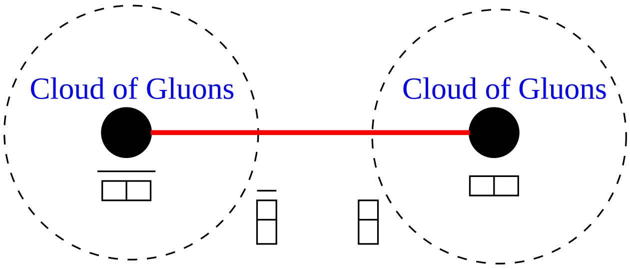
Outline

The outline of this short review is

- Basic properties of the k-string
- Field theory results
- MQCD and AdS/CFT results
- k-string width
- Lüscher term for the k-string
- $1/N$ corrections to the tension
- Discussion and open questions

Properties of the k-string

The basic property of the k-string is that its tension does not depend on the quark representation, but only on its N-ality, k. This is due to screening: A cloud of soft gluons should effectively transform the source from its representation to the k-antisymmetric representation.



Two comments are in order:

(i). Screening does not occur in pure YM in $d = 2$.

(ii). In $d > 2$ and large-N screening requires a long time. For example: the "adjoint string" does not break at infinite-N.
$\langle W_{\text{adjoint}} \rangle = \exp -2\sigma_f TL + \frac{1}{N^2} \exp -T/L$, hence $\Lambda T \sim \log N$. In other cases $\Lambda T \sim N^\alpha$.

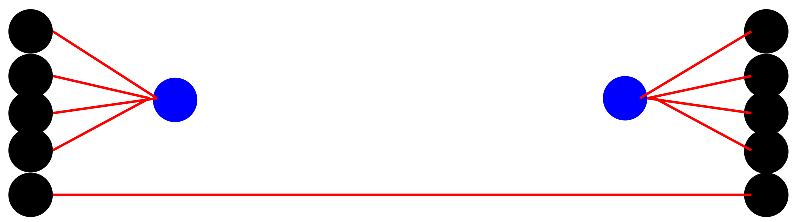
Properties of the k-string

A property of the k-string is the invariance $k \to N - k$. This is due to charge conjugation symmetry. k quarks in the antisymmetric representation are equivalent to $N - k$ antiquarks in the antisymmetric (bar) representation. In particular

$$\sigma_k = \sigma_{N-k}$$

Another property is $k \sim N + k$. Physically, if we place $N + k$ fundamental quarks close to each other, a baryon vertex will break N of them close to the source. Hence, in particular, $\sigma_{N+k} = \sigma_k$.

SU(4) with k=5



Properties of the k-string

Another property is that in the large-N limit $\sigma_k \to k\sigma_1$. The reason is that at infinite N there is no interaction between the fundamental strings. Hence if we bring the strings close to each other they will not form a bound state. Instead we will have a set of k non-interacting fundamental strings.

Later we will argue that the binding energy runs in powers of $1/N^2$, namely that $\sigma_k - k\sigma_1 = \mathcal{O}(1/N^2)$, for fixed k and large-N.

2d Field theory

It is easy to calculate the string tension in 2d pure YM theory, since the pure Yang-Mills theory is free. In the $A_1 = 0$ gauge the theory takes the form

$$S = \int d^2x \, -\frac{1}{2g^2}\mathrm{tr}(\partial_1 A_0)^2 \, ,$$

hence the potential is $V = Lg^2 T^a T^a$, namely the string tension is proportional to the quadratic Casimir

$$\sigma_R^{2d} = g^2 C_2(R).$$

Note that since the 2d theory is free there is no screening. For this reason the string tension is *representation dependent*. For example the adjoint string is stable and its tension is non-zero.

The 2d model inspired people to conjecture that in 3d and 4d the main effect of screening is to “reduce” the representation R to the k-antisymmetric and that the string tension in $d > 2$ is proportional to the quadratic Casimir

$$\sigma_k \sim C_2(\text{antisymmetric}) \sim k(N-k).$$

Or more precisely

$$\frac{\sigma_k}{\sigma_1} = \frac{k(N-k)}{N-1}.$$

This is called the “Casimir scaling hypothesis”.

Softly broken $\mathcal{N}=2$ super Yang-Mills

Douglas and Shenker considered the k-string in softly broken $\mathcal{N}=2$ super Yang-Mills theory. They used the Seiberg-Witten solution to calculate the k-string tension.

The result is

$$\sigma_k \sim m\Lambda N \sin(\pi \frac{k}{N})$$

The above formula is a first principle result in a 4d confining field theory!

Note that softly broken $\mathcal{N}=2$ SYM is confining, but the confining mechanism is different from what we expect in pure YM theory: $SU(N)$ is broken dynamically to $U(1)^{N-1}$, resulting in "Abelian confinement".

Note also the the QCD k-string in softly broken $\mathcal{N}=2$ SYM is a BPS object.

MQCD

Hanany, Strassler and Zaffaroni considered the k-string in MQCD. They considered a type IIA brane configuration that realizes $\mathcal{N} = 1$ super QCD and lifted it to M-theory to obtain a realization of the strongly coupled field theory. The resulting theory is "MQCD". It is a theory that preserves four supercharges and believed to be in the same "universality class" as SQCD. Namely, the interpolation between MQCD and SQCD is made by varying a parameter (x^{10}) and it is assumed that no phase transition occurs in this interpolation.

In MQCD the QCD-string is represented by a membrane that stretches in four-dimensions. It was shown that the membrane energy is $E = \sigma_k L$, such that

$$\frac{\sigma_k}{\sigma_{k'}} = \frac{\sin(\pi \frac{k}{N})}{\sin(\pi \frac{k'}{N})} .$$

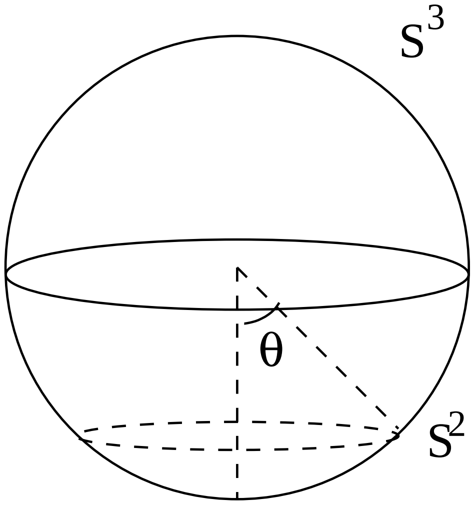
AdS/CFT

Klebanov and Herzog considered the k-string in the AdS/CFT framework. Since the supergravity approximation is valid at infinite N, their results hold when k/N is kept fixed.

They considered two similar (but different) backgrounds: Klebanov-Strassler and Maldacena-Nuñez. Both backgrounds describe in the IR the large-N limit of pure $\mathcal{N} = 1$ SYM (plus extra fields ...).

According to Klebanov and Herzog, when $k \sim N$ the k-string becomes a D3-brane. It is a four dimensional object. The D3 brane wraps an S^2 inside the $R^4 \times S^3$ geometry. So the four dimensional observer sees a two dimensional object. The tension of this object is determined by the angle θ of the S^2 inside the S^3.

AdS/CFT



Their result, exact for MN and approximate (93%) for KS, is that $\theta_k = \pi \frac{k}{N}$ and

$$\sigma_k = \sin \theta_k = \sin \pi \frac{k}{N} \, .$$

k-string width (quantum broadening)

Lüscher, Münster and Weisz calculated the width of the QCD-string, by placing a probe Wilson loop and measuring the decay of the electric flux transverse to the QCD-string as a function of the distance from the string. Similarly one may consider a circular large Wilson loop of radius R and a probe Wilson loop of radius r. The width is

$$\omega^2 = \frac{\int h^2 P(h) dh}{\int P(h) dh} = \frac{1}{2\pi\sigma} \log R/r \,,$$

(with $P(h)$ the connected Wilson loops two-point function) namely the string square width grows logarithmically with its size.

A measurement of k-string width by placing a Wilson loop of the same N-ality (a probe k-string) results in Armoni and Ridgway

$$\omega_k^2 = \frac{1}{2\pi\sigma_k} \log R/r \,,$$

This is expected since the k-string is simply a string with a tension σ_k.

Lüscher term of the k-string

Consider the short distance corrections to the inter quark potential. The leading non-trivial correction is due to Lüscher

$$V = \sigma L + \frac{\gamma}{L} \, ,$$

where $\gamma = -\frac{\pi}{24}(d-2)$. It corresponds to the fluctuations of the string in the transverse dimensions.

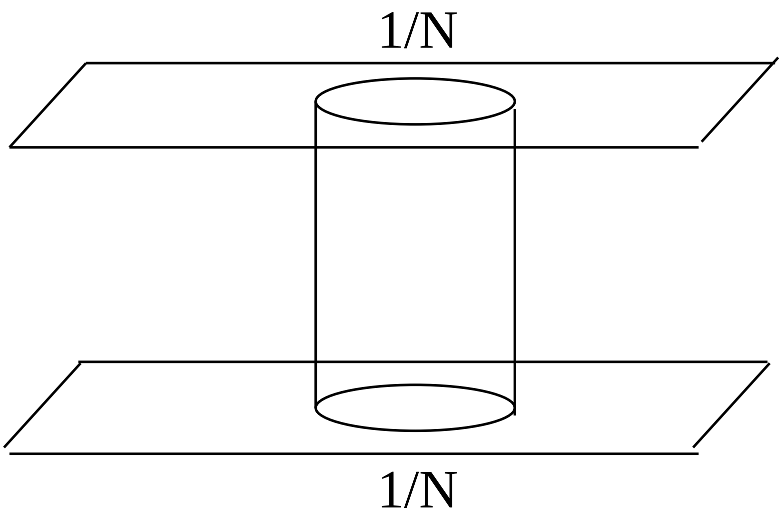
The generalization to the k-string case is expected to be

$$V_k = \sigma_k L + \frac{\gamma}{L} \, ,$$

with the same γ as in the case of the fundamental string. The reason is that, quite similar to the width issue, the k-string is simply a string whose tension is σ_k. Far away from the k-string, we should see a string described by a Nambu-Goto action.

$1/N$ corrections to the string tension

The k-string is formed by binding together k fundamental strings. The fundamental strings interact with each via an exchange of glueballs. The leading interaction is hence (Armoni and Shifman) $\mathcal{O}(1/N^2)$



Therefore for fixed k (say $k = 1$ or $k = 2$) we claim that

$$\sigma_k - k\sigma_1 = -\frac{C_k}{N^2} + \ldots .$$

In particular, the Casimir scaling hypothesis

$\frac{\sigma_k}{\sigma_1} = k - \frac{k^2}{N} + \ldots$

is inconsistent with an expansion in powers of $1/N^2$.

Summary/Discussion

Issues for discussion

- Corrections to the tension: $1/N^2$ or $1/N$?
- Size of lattice needed to measure the stable QCD string. To see the stable antisymmetric string should requrie a lattice whose size grows with N, $\sigma\mathcal{A} \sim N^2$. Did the current lattice simulations measure the stable k-string ?
- Width and Lüscher term of the k-string at finite temperature, where the k constituents are probed.